Minima, Maxima, and Inflection Points of Functions

Let’s first define a function. Note how Maple wants the exponential function input.

```maple
> f := x->x*exp(-x);
f := x → x \cdot e^{-x}
```

```maple
> plot(f(x), x=0..6);
```

First, we seek the maximum value of \( f(x) \), and \( x \) value at which this occurs.

```maple
> diff(f(x), x);
-\( e^{-x} - x \cdot e^{-x} \)

> solve( diff(f(x),x) = 0);
1
```

So, the maximum value occurs at \( x=1 \). This is certainly what it appears from the graph. The maximum value itself is \( f(1) \):

```maple
> f(1);
e^{-1}

> evalf(%);
0.3678794412
```

Now let’s find the inflection point. This is the point at which concavity changes -- i.e. when the second derivative is zero.

```maple
> diff( diff( f(x), x), x);
-2 \cdot e^{-x} + x \cdot e^{-x}

> solve( diff( diff( f(x), x), x) = 0 , x);
2
```

The inflection point occurs at \( x=2 \).
Let's plot the function and its first and second derivatives:

\[ \text{plot( } \{ f(x), \text{diff}(f(x),x), \text{diff( diff}(f(x),x),x)\}, x=0..6); \]

As expected, the first derivative is zero at the function's maximum, while the second derivative is zero at the inflection point.