Chapter 7.7: Improper Integrals

(What if the Cat Runs Forever?)

Calculus II

Spring 2021

College of the Atlantic

Consider the following two functions:

$$f(x) = \frac{1}{x^2}, \quad g(x) = \frac{1}{\sqrt{x}}. \tag{1}$$

1. Evaluate the following definite integrals:

$\int_1^{10} f(x)\,dx$	
$\int_1^{100} f(x)\,dx$	
$\int_1^{10000} f(x)\,dx$	
$\int_1^{100000} f(x)\,dx$	

2. What happens to $\int_1^b f(x)\,dx$ as b gets larger and larger?
3. Now, evaluate these definite integrals:

$\int_1^{10} g(x)\,dx$	
$\int_1^{100} g(x)\,dx$	
$\int_1^{10000} g(x)\,dx$	
$\int_1^{100000} g(x)\,dx$	

4. What happens to $\int_1^b g(x)\,dx$ as b gets larger and larger?
5. Why is your answer to 4 different than your answer to 2? Try sketching $f(x)$ and $g(x)$.

Improper Integral Practice

Evaluate the following improper integrals.

1. $$\int_0^\infty 3e^{-4x}\,dx$$

2. $$\int_0^\infty xe^{-2x}\,dx$$

   (The anti-derivative of xe^{-2x} is $-(1/4)e^{-2x}(2x+1)$.)

3. $$\int_0^\infty \sqrt{1+x^2}\,dx$$

4. $$\int_0^\infty e^{-x}\sin(x)\,dx$$

   The anti-derivative of $e^{-x}\sin(x)$ is $\frac{-e^{-x}}{2}(\sin(x)+\cos(x))$.

5. For what values of p does
   $$\int_1^\infty x^p\,dx$$
   diverge?

6. Does
   $$\int_1^\infty \frac{1}{x^3+2}\,dx$$
   converge? Answer this without evaluating the integral.