# Chapter 7.8: Comparing Improper Integrals

## Calculus II

## Spring 2021

### College of the Atlantic

1. Without using a calculator or computer, make rough sketches of the following functions, all on the same axes, for $x > 0$:

$$f_1(x) = \frac{1}{x^2} \ , \quad f_2(x) = \frac{1}{x} \ , \quad f_3(x) = \frac{1}{\sqrt{x}} \ . \tag{1}$$

2. Without using a calculator or computer, make rough sketches of the following functions, all on the same axes, for $x > 0$:

$$f_1(x) = e^{-x} \ , \quad f_2(x) = e^{x} \ , \quad f_3(x) = x^{-2x} \ , \quad f_4(x) = \frac{1}{x^2} \ . \tag{2}$$

3. Check your graphs using a calculator or computer.

4. Evaluate the following improper integrals:

$$\int_1^\infty \frac{1}{x^{1.1}} \, dx \ . \tag{3}$$

$$\int_1^\infty \frac{1}{x^{1.0}} \, dx \ . \tag{4}$$

$$\int_1^\infty \frac{1}{x^{0.9}} \, dx \ . \tag{5}$$

5. For what values of $p$ does

$$\int_1^\infty \frac{1}{x^p} \, dx$$

converge?

Determine whether or not the following integrals converge:

$$\int_2^\infty \frac{x^3}{x^4-1}\,dx \tag{6}$$

$$\int_1^\infty \frac{1}{x^2+7x+2}\,dx \tag{7}$$

$$\int_1^\infty \frac{3+\sin(3x^2)}{x^4+1}\,dx \tag{8}$$

$$\int_1^\infty \frac{\sin(x)+3}{\sqrt{x}}\,dx\ . \tag{9}$$

$$\int_1^\infty e^{-0.1x}\sin(x)\,dx\ . \tag{10}$$

$$\int_1^\infty \frac{2+e^{-x}}{x}\,dx\ . \tag{11}$$

For each of the following integrals, without calculating, determine if the integral is zero, positive, negative.

$$\int_\pi^\pi \sin(x)\,dx\ . \tag{12}$$

$$\int_{-\pi}^\pi \sin(x^2)\,dx\ . \tag{13}$$

$$\int_{-\pi}^\pi (\sin(x))^2\,dx\ . \tag{14}$$

$$\int_0^{2\pi} \sin(x)\,dx\ . \tag{15}$$

$$\int_0^{10\pi} \sin(x^2)\,dx\ . \tag{16}$$

Which of the integrals below equal 25, which are less than 25, and which are greater than 25? Answer by sketching the integrand.

$$\int_0^{25} e^x \, dx \ . \tag{17}$$

$$\int_0^{25} e^{-x} \, dx \ . \tag{18}$$

$$\int_0^{5} x \, dx \ . \tag{19}$$

$$\int_0^{5} 2x \, dx \ . \tag{20}$$

$$\int_0^{10} x \, dx \ . \tag{21}$$

$$\int_0^{10} x e^{-x} \, dx \ . \tag{22}$$