Class 25: Convergence of Series

Calculus II

College of the Atlantic. March 6, 2023

1. Do the following series converge? Experiment with python. Then figure out why the series either converges or diverges. Just focus on convergence and not the particular value the series converges to.

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \tag{1}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}} \tag{2}$$

$$\sum_{n=1}^{\infty} (-1)^{n+1}\sqrt{n} \tag{3}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^{1.1}} \tag{4}$$

2. Do the following series converge? Answer by comparing to a p-series or a geometric series. (But experiment with python if you wish.)

$$\sum_{n=1}^{\infty} \frac{1}{n^2} \tag{5}$$

$$\sum_{n=1}^{\infty} \frac{1}{n(n+4)} \tag{6}$$

$$\sum_{n=1}^{\infty} \frac{n+5}{n^3} \tag{7}$$

$$\sum_{n=1}^{\infty} \frac{n+5}{n^2+3} \tag{8}$$

$$\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n \tag{9}$$

3. Use the ratio test to see if the following series converge:

$$\sum_{n=1}^{\infty} \frac{1}{n!} \tag{10}$$

$$\sum_{n=1}^{\infty} \frac{n^2}{2^n} \tag{11}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2} \tag{12}$$

4. Consider the following power series:

$$S(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} . \tag{13}$$

Note that this infinite series is a function of x. We'll denote the partial sums by:

$$S_N(x) = \sum_{n=0}^{N} \frac{(-1)^n x^{2n}}{(2n)!} . \tag{14}$$

(a) For what values of x do you think will this series converge? Why do you feel this way?

(b) Write out the first few partial sums $S_N(s)$. Note that these are just friendly polynomials.

(c) Use the ratio test to check for convergence of the series. For what values of x does the series converge?

(d) Plot the first few partial sums from -10 to 10 or so. Do the functions start to look familiar?