Class 26: Power Series

Calculus II

College of the Atlantic. March 8, 2023

Consider the following power series:

$$S(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} . \tag{1}$$

We'll denote the partial sums by:

$$S_N(x) = \sum_{n=0}^{N} \frac{(-1)^n x^{2n}}{(2n)!} . \tag{2}$$

Note that $S_N(x)$ is just a polynomial. E.g.,

$$S_4(x) = 1 - \frac{1}{2}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 . \tag{3}$$

1. Compute a handful of partial sums for two different values of x:

	$x = 0.1$	$x = 2$
$S_2(x)$		
$S_{10}(x)$		
$S_{100}(x)$		
$S_{1000}(x)$		

2. Do $S_N(0.1)$ and $S_N(2)$ appear to be converging?

3. Let's check convergence using the ratio test. For what values of x does this series converge?

4. Instead of thinking of $S(x)$ one point at a time, we can think of it as a function of x and graph it. Make plots of $S_2(x)$, $S_5(x)$ and $S_{10}(x)$.

Another Power Series

Consider the following power series:

$$(x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \frac{(x-1)^5}{5} \cdots . \qquad (4)$$

As before we'll denote the partial sums by $S_N(x)$. Note that $S_N(x)$ is a polynomial. Write out the first several partial sums as a function of x:

$$S_0(x) = \qquad (5)$$

$$S_1(x) = \qquad (6)$$

$$S_2(x) = \qquad (7)$$

$$S_3(x) = \qquad (8)$$

1. Let's compute a handful of partial sums for two different values of x:

	$x = 0.5$	$x = 2.5$
$S_1(x)$		
$S_{10}(x)$		
$S_{100}(x)$		
$S_{1000}(x)$		

2. Do $S_N(0.1)$ and $S_N(2)$ appear to be converging?

3. Check convergence using the ratio test. For what values of x does $S(x)$ converge?

4. Instead of thinking of $S(x)$ one point at a time, we can think of it as a function of x and graph it. Make plots of $S_2(x)$, $S_5(x)$ and $S_{10}(x)$.