Class 27: Taylor Series

Calculus II

College of the Atlantic. March 8, 2023

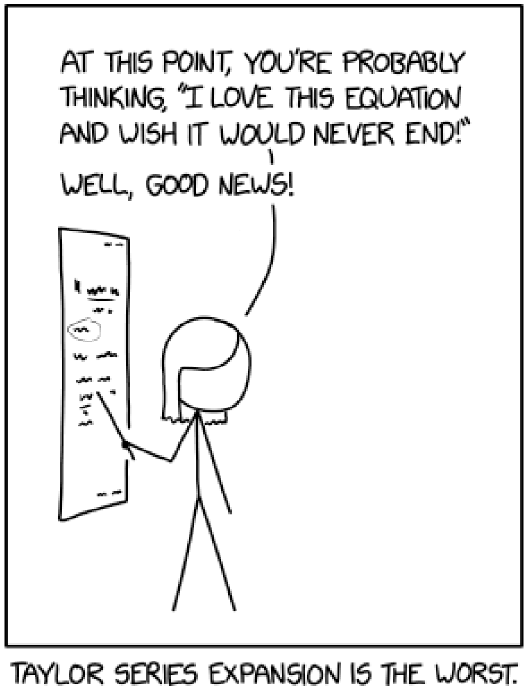

Figure 1: Taylor Series by Randall Munroe. Image source: https://xkcd.com/2605/.

Suppose you want to approximate the exponential function $f(x) = e^x$ by a polynomial. In this exercise we'll figure out how to build up the best possible approximate one term at a time. We'll call our approximate functions $P_N(x)$. We want our approximation to be as accurate as possible at the point $x = 0$.

0. We'll start by approximating the function $f(x)$ by a constant. This isn't going to be a very good approximation. But we have to start somewhere.

$$P_0(x) = C_0 . \quad (1)$$

What should we choose for the constant?

1. Now we'll approximate $f(x)$ by a line.

$$P_1(x) = C_0 + C_1 x . \quad (2)$$

We already know C_0. What should we choose for C_1, and why?

2. Next, we'll approximate $f(x)$ by a second-order polynomial:

$$P_2(x) \;=\; C_0 + C_1 x + C_2 x^2 \; . \tag{3}$$

   How should we determine C_2?

3. You can probably guess what's next. Let's approximate $f(x)$ by a third-order polynomial:

$$P_3(x) \;=\; C_0 + C_1 x + C_2 x^2 + C_3 x^3 \; . \tag{4}$$

   How should we determine C_3?

∞. At this point we likely see a pattern. Write down an infinite-order polynomial approximation for $f(x)$ using Σ notation.

1. Determine the Taylor Series for $\cos(x)$ about $x = 0$.
2. Determine the Taylor Series for $\ln(x)$ about $x = 1$.