Calculus II ~~Exam~~ Final Review Assignment

Due March 17, 2023

Thoughts on how to approach this assignment

- Here's a suggestion for how to do this assignment: Try these problems on your own, without or without notes. See how far you get. Then, if you have any questions, come to a help session and/or work with others in the class.
- That said, this is a normal homework assignment, in the sense that you are welcom me or Noelle.

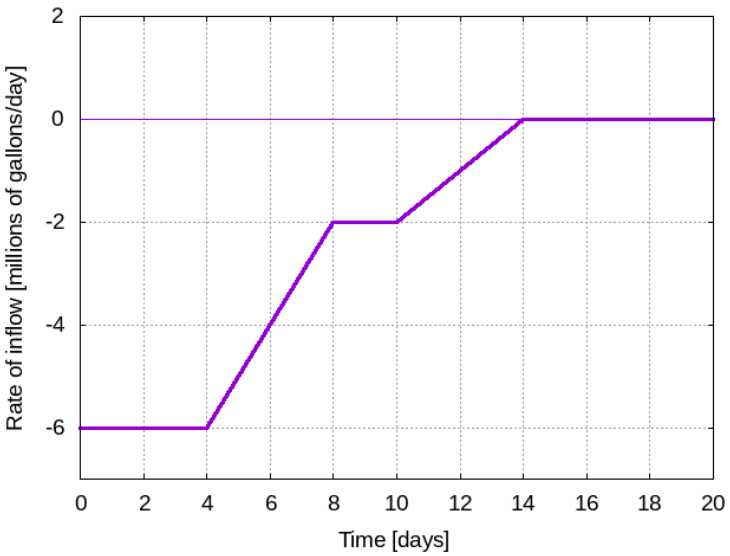

Figure 1: The rate at which water flows into the Kenneth C. Cline reservoir. (A negative rate means that water is flowing *out* of the reservoir.)

1. The Kenneth C. Cline reservoir is filled to capacity with 50 million gallons of water. The radical group CAHE[1] plants an explosive on the dam with the goal of removing the dam entirely. Unfortunately, they miscalculated[2] and the dam was only slightly damaged. Water leaked out of the dam fairly quickly for four days. Then repairs began and the leakage rate slowed. The rate at which water is leaving the dam is graphed in Fig. 1. Show your work or very briefly explain how you arrived at your answers.

   (a) What is the approximate volume of the water in the reservoir on day 8?

   (b) What is the approximate volume of the water in the reservoir on day 18?

[1] Coalition of Aging Heartfelt Environmentalists

[2] In one of their calculations they erroneously assumed that there are 2.2 kg in one pound.

Figure 2: A unicorn with wings. Image by brgfx on Freepik.

2. A population of unicorns on an island in Maine grows according to the function $r(t) = 5 + 2t^2$, where t is measured in months since January 1, 2023, and $r(t)$ has units of kg/month.

   (a) What is the total change in the unicorn biomass in the first two months. Determine approximate answers using left- and right-hand sums with $\Delta t = 0.5$.

   (b) Write a python program to determine a more accurate estimate for the total change in the unicorn biomass. Have your program break the time interval up into 1000 smaller time intervals. Please attach your colab notebook in google classroom.

   (c) Write the total biomass change as a definite integral, and use the fundamental theorem of calculus to evaluate the integral.

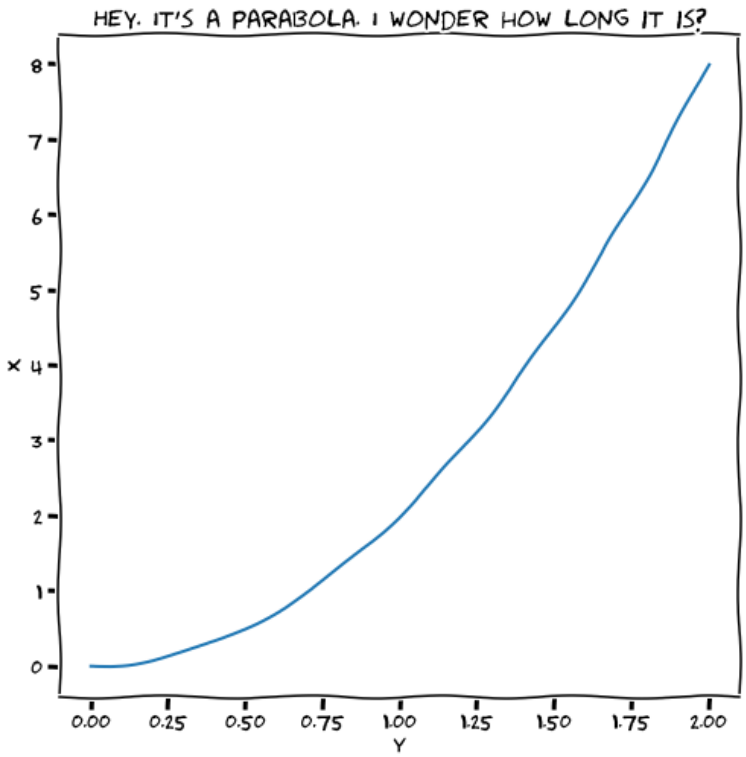

Figure 3: A parabola.

3. Figure 3 shows a parabola: $f(x) = ax^2$.

   (a) Determine the value of a.

   (b) Write down a definite integral that represents the arc length of the parabola.

   (c) Use wolfram alpha or python to evaluate the definite integral.

4. Consider the area bounded by the function $f(x) = e^x$, the line $y = 2$, and the y-axis.

   (a) Sketch this area

   (b) What is the volume of the shape formed if this area is rotated about the y-axis?

   (c) What is the volume of the shape formed if this area is rotated about the x-axis?

5. You are carrying out one of your cosmic ray unicorn generation experiments. An especially large cosmic ray hits your apparatus and instead of creating a unicorn, there is an explosion and suddenly the atmosphere is filled with unicorn dust. Your research team determines that the density of the unicorn dust in the air is well described by:

$$\rho(z) = 13/z^2 \,, \tag{1}$$

where z is measured in meters above the surface of the earth, and $\rho(z)$ has units of micrograms per cubic meter.

Your experimental facility is rectangular plot of land that is 10 meters wide and 25 meters long.

(a) What is the mass[3] of the unicorn dust above your experimental facility up to an altitude of 100 meters?

(b) What is the total mass of the unicorn dust exactly 55 meters above your experimental facility?

(c) What is the total mass of the all the unicorn dust above your experimental facility?

[3] If you can harvest enough of the unicorn dust, you could sell it to Tory's Donuts & and Pastries. They need unicorn dust to make one of their donuts.

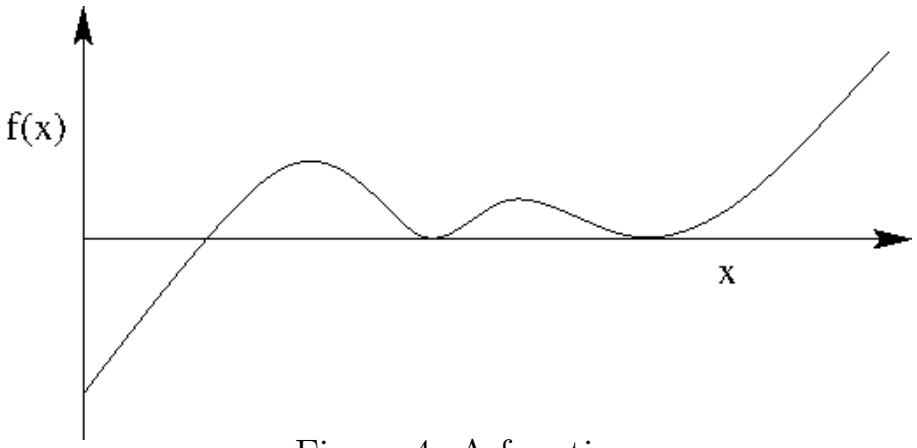

Figure 4: A function

6. Sketch

$$\int_0^t f(x)\,dx\ , \tag{2}$$

where $f(x)$ is shown in Fig .4.

7. Let

$$C(x)\ =\ \int_0^x \cos(t^2)\,dt\ . \tag{3}$$

What is $C'(x)$?

Figure 5: Three giraffes in Masai Mara National Park. Image by wikipedia user Byrdyak, licensed under the Creative Commons Attribution-Share Alike 4.0 International license. Image source: https://commons.m.wikimedia.org/wiki/File:Three_giraffes_01.jpg.

8. The distribution of the height of giraffes is well described by a normal distribution with a mean of 16 feet and a standard deviation of 2 feet. Use either wolfram alpha or z-tables to answer these questions.

   (a) If you select a giraffe at random, what is the probability that its height is more than 17 feet?

   (b) If you select a giraffe at random, what is the probability that its height is between 15 and 17 feet?

   (c) If you select a giraffe at random, what is the probability that its height is exactly 16 feet?

   (d) If you select a giraffe at random, what is the probability that its height is between 0 and ∞ feet?

9. You modify your unicorn-making apparatus so that it now makes giraffes. You run your experiment for a few months, and you end up with 100 giraffes. The average height of your giraffes is 16.5 feet.

   (a) Suppose you sampled the heights of 100 giraffes from the wild population. (The giraffe population described in the previous problem are wild giraffes.) The average of these 100 giraffes is a random variable; if you sampled another 100 giraffes you wouldn't get the same average. How is this random variable (the average of a sample of 100) distributed?

   (b) If you sampled 100 giraffes from the wild population, how likely is it that you could get an average height that is 16.4 feet or larger?

   (c) Do you think your lab-generated giraffes are significantly taller than the wild giraffes?