15.1–2: Optimization

(and also a problem about second derivatives from 14.7)

Calculus III

College of the Atlantic

1. Let the temperature in a metal rod be given by the function $T(x,t) = 100e^{-t}\sin(\pi x)$, where t is measured in minutes and x in meters. The rod is one meter long. (So $0 \leq x \leq 1$.)

   (a) Sketch $T(x,0)$ and $T(x,0.1)$.

   (b) Using the two sketches you just drew, determine the signs of f_x, f_t, f_{xx}, and f_{xt} at $x = 0.2$.

   (c) Using the two sketches you just drew, determine the signs of f_x, f_t, f_{xx}, and f_{xt} at $x = 0.5$.

   (d) Using the two sketches you just drew, determine the signs of f_x, f_t, f_{xx}, and f_{xt} at $x = 0.8$.

2. Consider the function $f(x,y) = 1/x + 1/y + xy$.

   (a) Use calculus and algebra to find any critical points.

   (b) By graphing the function and using common sense/critical thinking, classify any critical points.

   (c) Graph a close-up of the function near any critical points you found. What do the contour lines look like?

   (d) Graph a close-up of the function near $x = 2$, $y = 3$. What does the graph look like? What do the contour lines look like?

3. (Example 2 from Chapter 15.2, slightly modified.) Twenty cubic meters of tofu are to be delivered to College of the Atlantic. The tofu-maker plans to purchase an open-top box in which to transport the tofu in numerous trips. The total cost is the cost of the box plus \$2 per trip—perhaps this is bio-diesel needed to fuel her truck. The box must have a height of 0.5 meters but the tofu-maker can choose the length and the width. The cost of the box is \$20/m^2 for the ends and \$10/m^2 for the bottom and sides. What size box should the trucker buy to minimize the total cost?