18.3 and 18.4: Path-independent Integrals and Green's Theorem

Calculus III

College of the Atlantic

1. Consider the vector field $\vec{F} = 2x\hat{\imath} + 2y\hat{\jmath}$.

   (a) Evaluate the integral:
   $$\int_{C_1} \vec{F} \cdot d\vec{r}\,, \tag{1}$$
   where C_1 begins at $(0,0)$ and ends at $(2,3)$.

   (b) Now find a scalar function f such that $\vec{\nabla} f = \vec{F}$. Use f to evaluate the integral in Eq. (1).

   (c) Evaluate
   $$\oint_{C_2} \vec{F} \cdot d\vec{r}\,, \tag{2}$$
   where C_2 is a clockwise circle of radius 14 centered at $1, 2$.

2. Consider the vector field $\vec{F} = 2xy\hat{\imath} + xy\hat{\jmath}$. Find a scalar f such that $\vec{\nabla} f = \vec{F}$. Use f to evaluate
   $$\int_{C_1} \vec{F} \cdot d\vec{r}\,, \tag{3}$$
   where C_1 begins at $(0,0)$ and ends at $(2,2)$.

3. Which of the following are gradient vector fields?
   $$\vec{F} = 7\hat{\imath} - 4\hat{\jmath} \tag{4}$$
   $$\vec{F} = 7\hat{\imath} - 4x\hat{\jmath} \tag{5}$$
   $$\vec{F} = 7\hat{\imath} - 4y\hat{\jmath} \tag{6}$$
   $$\vec{F} = x\hat{\imath} - y\hat{\jmath} \tag{7}$$
   $$\vec{F} = x^3\hat{\imath} + \frac{1}{y}\hat{\jmath} \tag{8}$$
   $$\vec{F} = x\cos(y)\hat{\imath} - \frac{1}{2}x^2\sin(y)\hat{\jmath} \tag{9}$$