Homework Assignment One

Chaos and Complex Systems

Due Friday September 15, 2006.

Note: Please be sure to include a list of the references you consulted and any students you worked with.

1. A hockey league has 30 teams. During the season each team plays each other team exactly three times. How many total games are there in the season?

2. For the graph in Fig. 1, determine the following:

   (a) The adjacency matrix.

   (b) The degree k_i of each node.

   (c) The average degree $\langle k \rangle$.

   (d) The degree distribution $P(k)$.

   (e) The cluster coefficient C_i for each node.

   (f) The average cluster coefficient C.

   (g) The path length ℓ_{ij} between nodes:

      i. 1 and 3

      ii. 2 and 8

      iii. 2 and 4

      iv. 6 and 1

   In your responses, be sure to state clearly the definitions of all quantities you're calculating.

3. Consider a regular graph that is a five by five grid. Assume that the grid is wrapped around on itself so that the right most nodes are neighbors with the left most nodes, and the top nodes are neighbors with the bottom nodes. A picture of this graph is shown in Fig. 2. (This picture may or may not be the most useful way to think of this graph.) This shape turns out to be a donut. For this graph, do the following:

   (a) Write down the adjacency matrix.

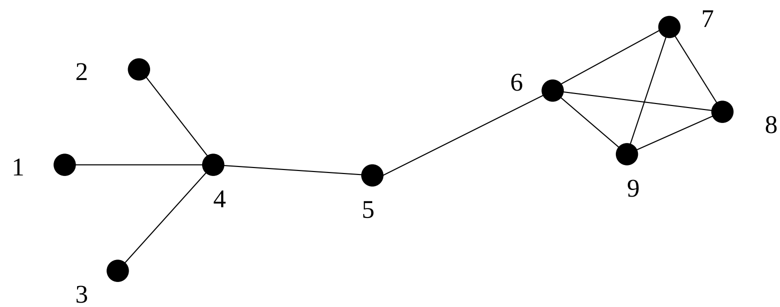

Figure 1: The network for problem 2.

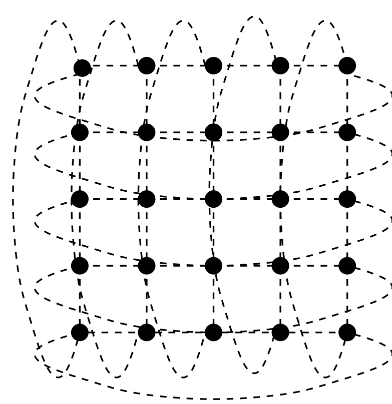

Figure 2: The network for problem 3.

(b) Calculate the average cluster coefficient.

(c) Calculate the diameter.

(d) Calculate the mean path length ℓ.

(e) Calculate the degree distribution.

(f) Calculate the average degree $\langle k \rangle$.

If this problem seems too easy, it probably means you're doing it correctly.

4. Now consider a regular graph like the one in the previous problem, but let the size be $N \times N$. Calculate:

   (a) The average cluster coefficient.

   (b) The degree distribution.

   (c) The average degree $\langle k \rangle$.

   (d) Calculate the diameter.

   (e) **Optional:** Calculate the mean path length ℓ.

5. **Optional. Do this if you like probability and/or want to mess around with limits.** In the Erdös-Rényi random graph model, the probability that a node has degree k is given by:

$$p_k = \binom{n}{k} p^k (1-p)^{n-k} \,. \tag{1}$$

Where n is the number of nodes in the graph. I asserted in class that as n gets large,

$$\binom{n}{k} p^k (1-p)^{n-k} = \frac{z^k e^{-z}}{k!} \,, \tag{2}$$

where $z \equiv p(n-1)$.

Show that Eq. (2) is true in limit of very large n and fixed k. This is a standard, but difficult calculation. It's not specific to random graphs but is a general result from probability theory. The left hand side is the binomial distribution and the right hand side is a Poisson distribution. To do this you'll probably need to consult other references. If you do so, be sure to cite your sources and explain your method thoroughly.