DAVID P. FELDMAN

SUSTAINABLE ENERGY: FOUNDATIONS AND SKILLS

PRINTED ON SEPTEMBER 25, 2018

PUBLISHED BY PRINTED ON SEPTEMBER 25, 2018

printed on september 25, 2018

http://energy.coa.edu

First printing, September 2018

Contents

Acknowledgments

This book began as a class that was developed with my former colleague Anna E. Demeo. I am grateful for her friendship, insight, humor, and enthusiasm. Anna is currently Associate Professor of Sustainable Design Engineering at the University of Prince Edward Island.

The collaboration between Anna and me was initially funded by the Maine Space Grant Consortium via their Research and Higher Education Program. I am grateful for their generous support at several stages of this project.

This text was produced exclusively with open-source, freely available software: the book was typeset using LaTeXand the tufte book class; figures were produced with inkscape and gimp, etc. I am also grateful for resources such as google scholar, google books, the zotero bibliography manager, and wikipedia.

Spencer Gray and Andrea Russel helped with procuring data.

Conversations with Jay Friedlander about different ways of valuing investments were extremely helpful as I put together Chapters 6 and 7. These sections also drew on a number of conversations I've had with Andy Griffiths over the years.

Doreen Stabinsky provided input on a number of issues, including agriculture and negative carbon technologies.

-1 Preface

The goal of this textbook is to give readers the knowledge and skills needed to be effective advocates for sustainable energy and to make wise choices among different options for renewable power generation and energy conservation. These skills and knowledge include basic understanding of energy and power as well as a facility with simple financial tools used to evaluate investments. But most important is a willingness to do basic calculations and estimates and use these numbers to think critically. In my experience these skills and habits of mind are accessible to almost all students, including those with only a modest preparation in mathematics.

This book is based on a class I have taught yearly for the last eight years at College of the Atlantic. The class was developed together by Anna Demeo and me, and we team-taught the class for a number of years. We originally used parts of David MacKay's *Sustainable Energy—Without Hot Air* (UIT Cambridge, 2008) for the course. I love this book: it is engaging, accessible, grounded, and MacKay does a masterful job of making potentially abstract quantities tangible and meaningful. But this book it was not intended to serve as an elementary textbook.

-1.1 Central Premises

There are several fundamental beliefs that inform my thinking about this book.

- **The climate problem is an energy problem.** Burning fossil fuels for the production of energy is responsible for the vast majority of the carbon dioxide. This carbon dioxide is the primary driver of climate change. Agriculture and deforestation are important, as well, but energy is by far the largest contributor. There is widespread agreement that avoiding catastrophic climate change must involve a transition almost entirely away from fossil fuels.

- **Solutions that scale.** Climate/energy is a big problem that requires big solutions. We produce and use energy on a large scale, and the vast majority of this energy derives from fossil fuels. Thus, to successfully prevent catastrophic climate change we must employ large solutions that move us away from fossil fuels. This requires solutions that scale up to address the magnitude of the issues we face. Learning how a residential wind turbine works is interesting, and learning how to build your own is a fun hobby. But understanding the land area and cost associated with large wind farms is essential for developing serious solutions to the climate/energy problem.

- **Energy numeracy.** I believe that energy and power units—the kilowatt-hour and the kilowatt—need to be as commonly understood as inches and miles, pounds and tons. Is forty miles too far to walk in a day? The answer to this question is clearly yes. Is forty kilowatt-hours a lot of energy to use in one day? I want readers to be able to answer questions about kilowatt hours as confidently as they can answer questions about miles.

- **Estimation.** A key part of working numerically is the ability to make quick numerical estimations, a skill that I find many students are surprisingly uncomfortable with. The text gives explicit instruction in estimation techniques and include multiple examples of back-of-the-envelope calculations.

- **Understanding large numbers.** Energy numbers tend to be huge and difficult to understand. Large numbers are often used, perhaps unintentionally, to obscure or intimidate. Megawatts, gigawatt-hours, and quadrillions of BTUs are all common units in energy and climate discussions. Large numbers also arise when considering the land mass needed for solar or wind or biomass: tens of thousands of square miles or millions of acres. This book will teach simple but important techniques for understanding large numbers and making them less abstract and more personal. I think this is absolutely essential for rational discussions about our energy and climate future.

- **Numbers not adjectives.** This is a mantra of MacKay in his book. Energy is an inherently quantitative issue. It is not enough to say that there is huge potential in solar power. How huge? And how much area would the solar cells take? It's not enough to say that insulating your attic will save a lot of energy. How much, and how does that investment compare to investing in, for example, solar panels? How much less greenhouse gas emissions would you be responsible for if you got a more fuel efficient car, and how would this deduction compare to shifting to a vegetarian diet?

- **Just a bit of physics.** There are a relatively small handful of basic notions from physics that we think are required for understanding sustainable energy. These topics include the law of conservation of energy, the difference between power and energy, and some basic formulas for kinetic energy, thermal energy, and so on. I construct a foundation in these areas, assuming essentially no prior knowledge of physics. I then use this foundation to build an understanding of some key results, such as the fact that the power in wind of speed v is proportional to v^3, and that the amount of energy needed to heat a home is directly proportional to the difference in temperature between the inside and the outside. Throughout our book the central goal is to prepare readers to understand and do effective work in sustainable energy. The physics we introduce is in service of this goal.

- **Finance.** It is fairly easy to come up with good ideas for generating energy or reducing energy use. Usually the hard part is securing funding for a project. In this area some basic knowledge of financial mathematics goes a very long way. Understanding the vocabulary of finance is essential for conversation with banks and other funders, something that is necessary for all but the smallest projects. Additionally, financial metrics give one a tool to use when choosing among different potential investments.

- **Choosing among options.** There are a plethora of viable ways to use less energy or to generate energy without fossil fuels. However, given the reality of limited time and resources, it can be a challenge to choose among these many options. The initial cost, as well as the payback period, factor into these choices as do numerous other considerations, including community support (or lack thereof), local ordinances, state regulations, and utility rules. How does insulating my house compare to putting up solar panels on my roof? Should a state government spend funds to help homeowners insulate their homes, or should it invest in public transportation?

- **You can't stop reality from being real.** Or, as Richard Feynman puts it, "You can't fool mother nature." A plan to get the world off fossil fuels has to be grounded in reality, not sentiment. The approach in this book is data driven and considers bounds on energy use and production arising from the law of conservation of energy and other constraints.

- **The truth.** The situation around climate change looks bleak and there are enormous challenges before us. We are confident that any solution has to begin with an honest accounting of our current situation. Downplaying the scale of the problem or ignoring it entirely will not fix anything. However, I have found both a sense of empowerment and a measure of comfort come out of facing reality head on. Instead of getting discouraged, let's get to work thinking about solutions that add up and that are based on actual numbers and not wishful thinking.

- **Fun.** Just because the situation is a bit daunting doesn't mean one can't have fun. We really like figuring this stuff out and sharing fun and creative ways of thinking of things. And I also like good (and bad) jokes. There's a lot of hard work to do, so we might as well do it with smiles on our faces.

-1.2 *What this book isn't*

To get a better sense of the scope of our book, it might be helpful to say a bit about what this book is not about:

- Energy policy
- Social and environmental impacts of energy technologies
- How to weather-proof your house
- How to build your own wind turbine or install solar cells
- The science of climate change

While I do cover these topics at length, this book should help readers think about about the items above list in a deeper and more grounded way. I'll include ample references for readers who wish to delve more deeply.

-1.3 *Audience*

This text covers the basic knowledge and analytical skills needed by almost anyone working in sustainable energy. The topics in this book will be of interest to students focusing in sustainability and environmental studies, as well as those whose primary interests are business, energy policy, or physics. Additionally, the material should prove valuable to solar PV installers, home insulators, business owners interested in saving money and greening their operations, climate change activists, planning board members, state representatives, and anyone interested in getting involved in a community renewable energy project.

This book should be well suited for readers with a range of backgrounds. At a minimum, readers should be comfortable with basic math and very simple algebra. Some facility with units and unit conversions, at the level one encounters in a high school chemistry or physical science class, is certainly helpful, but not strictly necessary. We review unit conversions in the book.

This text is intended to be used in a classroom setting. The format of the book encourages flexibility and should enable it to be adopted for courses with different emphases. I see this book as a good match for an interdisciplinary, foundational sustainability course in an environmental studies program, a business program, and perhaps also in masters programs emphasizing sustainability. While not a comprehensive physics of energy textbook, there is nevertheless enough physics content that it might also be used as a text for a physics of energy course, although likely the instructor would need to supplement the course with a bit more advanced material.

-1.4 *Notes on the Book's Organization*

The book is divided into five parts.

I. Part one provides some perspectives and framing for the numbers and analysis that will come. We aim to give a bigger-picture preview and survey before we dive into details. As part of this, we introduce readers to tons of CO_2 per person per year and kWh per person per day as key numerical indicators.

II. This covers some basic physics and some vocabulary and concepts that we think are fundamental. All readers, regardless of their particular interests, need to be comfortable with the majority of topics from Part II.

III. Here we introduce the time value of money and several standard ways of valuing and comparing investments: payback time, return on investment (ROI), and the internal rate of return (IRR). We think this material is important and fun, but it can be omitted if needed. It can also be highlighted and applied to each of the chapters in part IV.

IV. In this part we cover a large number of topics. These include ways of sustainably generating energy and the main uses of energy: heating, lighting, transportation, growing food, etc. These chapters are intended to be more or less independent of each other. They could be covered in almost any order. In fact, some of these chapters could be covered immediately after reading their corresponding chapter in Part II. For example, Chapters 10 and 11 could be covered immediately after Chapter 5. This is how we sequence this material when we teach our class at COA.

V. The chapters in the Appendix go over some basic skills that could be covered at any time. More mathematically experienced readers may not need to consult these chapters, which is why we have put them in the appendix. The appendices also include a chapter that compiles useful data for easy reference.

There is far more material in this book than can be taught in a typical one-term college course. The structure of the course is sufficiently flexible that instructors should be able to choose a set of topics that works for their goals and their students' backgrounds.

Part I

Basics

0

Opening Remarks

This chapter could pretty closely follow McKay's opening chapter(s) MacKay (2009).

0.1 Climate Change

0.2 Other Reasons to Care about Energy

1. Saving money

2. Geopolitics

3. Not peak oil. We will be doomed long before all the oil runs out. Need citations. There is not quite 100% agreement on just how much oil is left and how hard/easy it would be to get it.

4. Environment. Fossil fuels, especially coal, produce ozone and particulate matter that is fantastically unhealthy. Look up some estimates for deaths due to asthma. Both in US and developing world, if possible.

0.3 Measuring Greenhouse Gas Emissions

1. Discuss CO_{2e} vs. CO_2. Carbon dioxide is not the only gas that causes global warming. Other gases warm the climate as well, most significantly methane (CH_4) and Nitrous Oxide (NO_2). These gases are more powerful greenhouse gas agents than carbon dioxide. For example, on a per-molecule basis, methane is about 25 times more powerful a greenhouse gas than carbon dioxide, and nitrous oxide is almost three hundred times more powerful (Solomon et al., 2007, Table 2.14, p. 212). There are other greenhouse gases in addition to methan and nitrous oxide.

   Often non carbon dioxide greenhouse gases are "converted" to what is called carbon dioxide equivalent, abbreviated CO_{2e}.

2. Carbon vs. Carbon dioxide
3. Carbon intensities. These are CO_{2e} normalized by dividing by something such as a country's GDP or total energy output.
4. Current global emissions and country emissions. Make nice graphs.
5. Historical emissions.
6. "Atmospheric space."

0.4 Some Current Data

Here is 2013 emissions data. This is in units of Gt CO_{2e}. This does not include land use and forestry.[1]

[1] All data is from the World Resources Institute's CAIT Climate Data Explorer tool at http://cait.wri.org. This data has migrated to http://www.climatewatchdata.org.

- World: 45 Gt
- China: 12 Gt
- USA: 6.2 Gt
- India: 2.9 Gt
- European Union (28 countries): 4.2 Gt
- Germany: 0.89 Gt
- United Kingdom: 0.54 Gt
- Canada: 0.74 Gt
- Mexico 0.73 Gt
- Thailand 0.37 Gt
- Philippines: 0.17 Gt
- Chile: 0.10 Gt
- Bolivia: 0.046 Gt
- Kenya 0.060
- Rwanda: 0.0066 Gt

And here are some 2013 populations, in millions of people:

- World: 7,180
- China: 1,360
- India: 1,280
- USA: 316

- European Union (28 countries): 503
- Germany: 82.1
- United Kingdom: 64.1
- Canada: 35.2
- Mexico 124
- Thailand 67.5
- Philippines: 97.6
- Chile: 17.6
- Bolivia: 10.4
- Kenya: 43.7
- Rwanda: 11.1

These data are also from the WRI CAIT Climate Data Explorer.

These are some very big numbers. But happily, it turns out that they are similarly big. For example, the worldwide emission of CO_{2e}per year is 45 Gt. But the worldwide population is 7180 million people, which happens to equal 7.18 Gp, where I've invented the unit of gigapeople: 1 Gp= 1000 million people =1 billion people. So the average CO_{2e}emissions per person worldwide is

$$\frac{45\,\text{Gt}}{7.180\,\text{Gp}} \approx 6.3\frac{\text{Gt}}{\text{Gp}}\,. \tag{1}$$

But the "gigas" cancel, so we end up with an important number to remember: the average CO_{2e}emissions per year is 6.3 tonnes per person. What is the average $CO_{2e}$of someone from the US? A calculation similar to that which we did for the world yields 19.6 tonnes of CO_{2e}per person per year. So we see that the average per person emissions for the US is about 3 times the world average. I'll usually round this figure up to 20.

The average yearly CO_{2e}emissions for China is 8.8 tonnes per person, and for India it's 2.3 tonnes per person, about a third of the worldwide average. For Kenya and Rwanda, the CO_{2e}emissions per year are: 1.4 and 0.6 tonnes per person.

0.5 *Eliminating Fossil Fuels*

So, it's best to think about eliminating fossil fuels entirely. This basically means using non-fossil fuel sources for all our energy needs. There are two ways to do this.

1. Reduce worldwide energy use.

2. Replace fossil fuels energy with non-fossil fuel energy: solar, hydro, wind, nuclear, etc.

What will it take to do this? Is it even possible? To begin answering this we need to know how much energy is currently used. Here are some numbers to start us off:[2]

[2] Get some Data from EIA? It's not clear where MacKay got his data. He says UNDP, but I haven't been able to find it.

0.6 Alternative vs. Renewable vs. Sustainable

Some remarks along the lines of (Pagnoni and Roche, 2015, Section 2.2)

1
Energy

1.1 Jane's Blocks

This is an example from Section 4-1 of Richard Feynman's famous introductory lectures on physics (1977). I'm going to write this section later. In the meantime, if you want to read Feynman's original version of the story, you can do so at the following url: http://www.ioc.ee/~silvio/nrg/feynman_parable.pdf

1.2 Conservation of Energy

So the main point is that energy is this thing—an abstract quantity—that stays the same no matter what. It is like Jane's blocks, except there are no blocks. Energy is always hidden. It is something that is measured indirectly. There are a number of different forms of energy, many of which you are no doubt familiar with:

- Kinetic. This is the energy associated with motion.
- Gravitational. This is the energy associated with an object's change in altitude. For example, an object on top of a table has more gravitational potential energy than when that object is on the floor.
- Thermal. This is the "hidden" kinetic energy associated with the motion and vibration of molecules.
- Chemical. This is the energy stored in chemical bonds. When we eat food, we take the chemical energy in the food and convert it into heat and kinetic energy. Similarly, burning oil or wood converts chemical energy into heat.
- Radiant. This is the energy associated with electromagnetic radiation. This includes light, X-rays, UV radiation, and so on. Energy from the sun arrives on Earth as radiant energy.

- Nuclear. This is the energy stored in nuclear bonds—the forces that hold the nucleus of an atom together.

Just as was the case for Jane's blocks, each of these types of energy has a different formula associated with it. The above list is not exhaustive, but does enumerate the main types of energy we'll be working with.

As objects in the universe interact, they exchange energy. But energy is never created or destroyed; it just changes form. For example, Anna might drop her cell phone. When she does, the phone falls downward and the gravitational energy of the phone is converted to kinetic energy as it falls. The phone hits the floor, bounces a few times, and comes to rest. The kinetic energy has now been converted to thermal energy in the floor and the phone; their molecules are now vibrating a tiny bit more quickly than they were before the collision. Anna picks up her phone and puts it back in her pocket. This increases the gravitational energy of the phone. To do so, Anna must use some of the chemical energy stored in her body. She can replace this energy later when she eats lunch. And so on.

In this sense conservation of energy may be thought of as a sort of cosmic accounting. It is a form of bookkeeping. Accounting is a way of keeping track of financial transactions, but it does not speak to why those transactions have occurred. We will use the conservation of energy similarly: we will use it to keep track of energy as it changes from one form to another. This view of physics is not quite mechanistic, in that it focus neither on the forces that make objects move nor the details of the interactions between objects. Nevertheless, conservation of energy is one of the most powerful ideas in physics, and is the key to understanding the physics behind sustainable (and not sustainable) energy.

1.3 Kinetic Energy and the Joule

There are many different types of energy, just as there are many different places Jane can hide her blocks. Each type of energy has a different formula associated with it. The first type of energy we will consider is the energy associated with a moving object. This type of energy is called *kinetic energy*. The kinetic energy of an object with a mass m and a speed v is given by:[1]

$$E_k = \frac{1}{2}mv^2 \,. \tag{1.1}$$

For example, a 2kg rock moving at 3 m/s has a kinetic energy of

$$E_k = \frac{1}{2}(2\text{kg})(3\text{m/s})^2 = 9\,\text{kgm}^2/\text{s}^2 \,. \tag{1.2}$$

So we see that the units of energy are kilograms-meters-squared-per-second-squared. This is a mouthful. Happily, this awkward unit has a

[1] For the purposes of this book, we will view Eq. (1.1) as fundamental. I.e., a fact of nature that need not be derived—it is just the way the world is. (Do I want to say anything more along these lines?)

name, so we don't have to use this long phrase. The unit is known as the *joule* and abbreviated simply as J. I.e., one Joule is defined as:

$$1\,\text{joule} = \text{kgm}^2/\text{s}^2 \,. \tag{1.3}$$

How much is a joule of energy?

On a human scale, not much. To paraphrase an example from Moore (2002), one joule is the kinetic energy of a two-liter bottle soda walking down the street at one meter per second.[2] If this two-liter soda bottle walked into you at the speed of one meter per second, it would not do much damage.[3] You would notice this, but it wouldn't hurt. The point is that a joule is a small unit of energy. If you left a typical toaster on for two minutes it would use $432,000,000$ joules. Raising the temperature of two kilograms of liquid water by 10 degrees Celsius would take $83,600$ joules.

[2] One liter of water has a mass of two kilograms. A speed of 1 meter per second is roughly the walking speed of a typical human.

[3] It would be really weird, because bottles of soda don't have legs and can't walk. But the collision wouldn't be physically harmful to you.

Thus, while the Joule is the standard unit for energy, it is too small for most human-scale interactions. So we seek a larger unit—one that will be easier to work with. We will, in fact, be led to several such units, since different types of energy are traditionally measured using different units. This is unfortunate, since it obscures the fact that energy is energy and that all types of energy are interchangeable. Moreover, some of these units are quite awkward.[4] But this is the world we live in, so we're going to have to become comfortable with multiple units. We will encounter our first commonly used, human-scale energy unit, the kilowatt hour, in Sec. 1.6. First, though, we need to introduce the crucial concept of power.

[4] We're looking at you, British Thermal Units (BTUs).

1.4 *But what* **is** *energy??*

So far we've said that energy is this thing that is conserved—that stays the same no matter what.[5] In so doing, we've sidestepped the question of what energy is. It turns out that this is actually a deep question. Energy is an abstract quantity that does not have a single, direct definition. That said, one way to make energy more concrete is to think of energy as the ability to do work.

[5] I'm not sure where this section goes. It also shouldn't be too long.

Work in physics is force exerted through distance. If you push a couch along the floor for two meters, the work you have done is equal to the force you exerted on the couch times the distance you pushed it.[6] The units of work are the same as that of energy: Joules in SI.

[6] Specifically, the work W is given by $W = \vec{F} \cdot \vec{dr}$, where $\vec{F}$ is the force vector and $\vec{dr}$ is the distance though which the object was pushed. The force and displacement are vectors, meaning we need to account for their direction as well as their magnitude. And "·" is the dot product, a way of multiplying two vectors that, roughly speaking, multiplies only the parts of the two vectors that are pulling in the same direction. We won't use this formula, but I include it since some of you likely have encountered this in past physics classes.

Energy, then, is the ability to do work. So an object that has a energy of 100 j has the ability to do 100 j of work. Conversely, if one does 100 j of work something, it now has 100 j of energy.

Energy is an abstract and powerful concept that provides a framework for thinking about a vast array of physical phenomena. This realization formed gradually over the 1900s. Finish this paragraph.

1.5 Power

Energy is transformed from one type to another. The *rate* at which this transformation occurs in known as *power*. Just like current is the amount of charge that flows per unit time, power is the amount of energy that flows per unit time:

$$\text{Power} = \frac{\text{Energy}}{\text{time}} . \tag{1.4}$$

The SI units for power are *watts*, abbreviated W, and defined as:

$$1\,\text{W} = 1\frac{\text{J}}{\text{s}} . \tag{1.5}$$

A Watt is a reasonable unit of power to describe many everyday phenomena. A typical light bulb might draw 40 watts and a toaster or a hairdryer around 1000 watts. In some contexts—thinking about how much power a town uses or a power station generates—larger units are more convenient, so we will often us kW (kilowatts) and MW (megawatts).[7]

To get a feel for power and Watts, let's work through a few examples:

[7] As with all kilo- and mega- units, 1 kW = 1000 W, and 1 MW = 1000 kW = 1,000,000 W.

Example 1.1. *In order to bring a pint of water to a boil to make coffee, a heating element transfers* 168,000 *J to the water in* 4 *minutes. What is the power associated with this energy flow?*
We start with the definition of power $P = E/t$:

$$P = \frac{168,000\,\text{J}}{4\,\text{min}} \left(\frac{1\,\text{min}}{60\,\text{s}}\right) = 700\frac{\text{J}}{\text{s}} = 700\,\text{W} . \tag{1.6}$$

Example 1.2. *A power of* 1 *kW flows for* 1 *hour. How many joules of energy is this?*
Since $P = E/t$, it follows that $E = Pt$. Since 1 kW is 1000 W, and since one watt is one joule per second:

$$E = 1000\frac{\text{J}}{\text{s}}(1\,\text{h}) \left(\frac{60\,\text{min}}{1\,\text{h}}\right) \left(\frac{60\,\text{s}}{1\,\text{min}}\right) = 3600000\,\text{J} . \tag{1.7}$$

Or, converting to MJ:

$$E = 3600000\,\text{J} \left(\frac{1\,\text{MJ}}{1,000,000\,\text{J}}\right) = 3.6\,\text{MJ} . \tag{1.8}$$

Example 1.3. *A hard-throwing baseball pitcher might throw over the course of a game* 100 *pitches at an average speed of* 90 *miles an hour. The mass of a baseball is* 145 *grams. If the pitcher throws these* 100 *pitches in one hour, what power has he delivered?*

Let's start by calculating the kinetic energy the pitcher gives to a baseball each pitch. To do so, we first convert miles per hour to meters per second:

$$90\text{mi/hr} = 90\text{mi/hr}\left(\frac{1\text{m/s}}{2.24\text{mi/h}}\right) = 40.2\text{m/s}\,. \tag{1.9}$$

(The conversion factor 1 m/s = 2.24 mi/h is a useful one. It is listed in Appendix D for easy reference.) A 145 g ball is 0.145 kg. We can then determine the kinetic energy of the thrown ball:

$$E = \frac{1}{2}mv^2 = \frac{1}{2}(0.145\,\text{kg})(40.2\text{m/s})^2 = 117\,\text{J}\,. \tag{1.10}$$

Power is energy/time. There are 100 such pitches in one hour, so:

$$P = \frac{100 \times 117\,\text{J}}{1\,\text{h}}\left(\frac{1\,\text{h}}{60\,\text{min}}\right)\left(\frac{1\,\text{min}}{60\,\text{s}}\right) = 3.25\,\text{J/s} = 3.25\,\text{W}\,. \tag{1.11}$$

1.6 *Kilowatt Hours*

While Joules are the energy unit of choice for physicists and chemists, they are not used much in the energy field. Instead, one of the most common units are *kilowatt hours*, abbreviated kWh. A kilowatt hour is the amount of energy that has flowed if one kilowatt flows for one hour. You have already seen kWh—that's what Example 1.2 was about. There you showed that one kWh is 3.6 million joules of energy.

A kilowatt hour is a unit of energy, just like an amp hour is a unit of charge. A kilowatt hour is the amount of energy that has flowed if one kilowatt (1000 J/s) flows for one hour. An amp hour is the amount of current if one amp (1 C/s) flows for one hour. In both cases we are multiplying a flow rate by a time.

Some information that I might turn into paragraphs at a later date:

- In Maine our local utility Emera sells electric energy to homes. Emera charges 17.2 cents for one kWh or electricity.[8]

- in the U.S., on average, one kWh of electricity production results in 613 grams of CO_2 being released into the atmosphere. The values for other countries can be found on page 335 of MacKay (2009).

- Worldwide averages for the carbon intensity of different modes of electricity generation can be found in Section D.3.

- The average Maine home uses 520 kWh of electricity per month.

- The average US home uses 900 kWh per month. (`http://www.eia.gov/tools/faqs/faq.cfm?id=97&t=3`.)

[8] Emera charges different rates for commercial customers. This is the topic of Section 3.2.

- The average home in Spain uses 344 kWh/month. The average home in Mexico uses 150 kWh/month. (http://shrinkthatfootprint.com/average-household-electricity-consumption)

Example 1.4. *A 3 kW electric pump runs for 4 hours a day. How much energy, in kWh, does the pump use?*
Energy is power times time. So

$$E = (3\,\mathrm{kW})(4\,\mathrm{h}) = 12\,\mathrm{kWh}\,. \tag{1.12}$$

Note that kW are units of power and kWh are units of energy.

Example 1.5. *A 800 W electric heater is on for 40 min. How much energy has it used? Answer in both kWh and J. How much would this cost in Maine?*
To end up in kWh, we'd like power in kW and time in hours. Let's convert:

$$800\,\mathrm{W} = 800\,\mathrm{W}\left(\frac{1\,\mathrm{kW}}{1000\,\mathrm{W}}\right) = 0.8\,\mathrm{kW}\,. \tag{1.13}$$

$$40\,\mathrm{min} = 40\,\mathrm{min}\left(\frac{1\,\mathrm{h}}{60\,\mathrm{min}}\right) \approx 0.67\,\mathrm{h}\,. \tag{1.14}$$

Energy is power times time, so:

$$E = (0.8\,\mathrm{kW})(0.67\,\mathrm{h}) = 0.536\,\mathrm{kWh}\,. \tag{1.15}$$

So this heater uses a bit more than half a kWh or energy. How many joules is this? Doing the conversion, we obtain:

$$0.536\,\mathrm{kWh} = \frac{3.6\,\mathrm{MJ}}{1\,\mathrm{kWh}} = 1.93\,\mathrm{MJ}\,. \tag{1.16}$$

So the heater will have used 1.93 million joules.

In Maine, electricity costs 17.2 cents for one kWh. So,

$$\mathrm{Cost} = (0.536\,\mathrm{kWh})\left(\frac{0.172\$}{1\,\mathrm{kWh}}\right) = 0.092\$\,. \tag{1.17}$$

or a bit more than nine cents.

1.7 *Energy Quality*

When physicists (and engineers?) say that energy is conserved, we mean that energy is neither created nor destroyed but just changes forms. Given this, what does it mean to conserve energy, as as environmentalists and others admonish us to do? What is there to conserve? After all, the total amount of energy always stays the same.

However, while the total amount of energy stays the same when converted from one form to another, it is not always easy to or feasible to turn the energy back into its original form. For example, one might burn some oil, converting chemical energy into thermal energy. This is easy to do. One just needs some oil and a match. Light the oil on fire. It burns. Things heat up. But it is much, much, much harder to take heat unburn it and end up with oil again.

So at issue is the *quality* of the energy. Talk about high vs. low quality energy. High quality is orderly energy and low-quality is disorderly. Etc.

1.8 Exercises

Exercise *1.1:* A 0.8 kg bird flies at 2 m/s. What is its kinetic energy?

Exercise *1.2:* A 120 pound hockey player skates at 8 m/s. What is her kinetic energy?

Exercise *1.3:* A 80 kg football player runs at 5 m/s. What is his kinetic energy?

Exercise *1.4:* A 1 pound brick flies through a congressman's window at 12 miles per hour. What is the brick's kinetic energy? Express your answer in both J and kWh.

Exercise *1.5:* In order to dry out some sheetrock "mud", I left a 1500 W heater on for two full days this weekend. How much energy did this use? Answer in both kWh and Joules. How much would this cost in Maine?

Exercise *1.6:* Convert the following to kWh:

1. 1 Joule
2. 1,000,000 J.
3. 7,500 J.

Exercise *1.7:* Convert the following to Joules:

1. 100 kWh
2. 31 kWh
3. 57,160 kWh

Exercise *1.8:* Water is pumped into a tank at the rate of 120 gallons/sec. How much water flows into the tank in one minute? How much in one hour?

Exercise *1.9:* A certain light bulb draws 120 W of power. How much energy does the light bulb use in one minute? How much does it use in one hour? Answer in both Joules and kWh.

Exercise *1.10:* Which of the following are units of energy? Which are units of power? Which units don't make sense?

1. kW
2. kWh
3. Joules
4. kW/h
5. MW
6. kWh/day

Exercise *1.11:* "How much power did you use this month?" Why does this question not make sense?

Exercise *1.12:* The lights in Dave's office draw 120 W. Suppose he leaves them on for three hours a day for a month. How much energy does this use? Express your answer in both kWh and Joules

Exercise *1.13:* If something generates 4.8 kW for one year, how much energy is this? Express your answer in kWh and MWh.

Exercise *1.14:* Suppose you leave a 1000W toaster on for an entire year.

1. How much energy does this use? Express your answer in both kWh and MJ.
2. How much does this cost per month in Maine?
3. Express the power of the toaster in kWh/day.

Exercise *1.15:* Water flows into a reservoir at the rate of 20 gallons/sec. How much water flows into the tank in three hours?

Exercise *1.16:* An appliance draws 20 watts. How much energy does the appliance use in three hours? Express your answer in both kWh and J.

Exercise *1.17:* What wattage light bulb uses 1 kWh in one day?

Exercise *1.18:* Suppose that in a typical day a typical person typically eats around 2500 calories of food.[9] These are dietary calories.

[9] The recommended daily caloric intake, according to the U.K. National Health Service is 2000 calories for women and 2500 calories for men (Service). Add citation to wikipedia about reality.

Confusingly, 1 dietary calorie equals 1000 "real" calories. (See Appendix D.)

1. How many Joules does a typical person consume in a day?
2. What power is this? Express your answer in kW.
3. Most of the food energy you consume ultimately gets converted to heat. Thus, we can view people as heaters—they convert chemical food energy into thermal energy. How many people would you need to have in a room to have a heating power roughly equivalent to one 1500 W space heater?

Exercise *1.19:* In the field of hydrology, water flowing into a reservoir at a rate of 1000 gal/s is known as a *kiloCline*.[10] If 2 kiloClines flow into an initially empty reservoir for 3 hours, how much water is in the reservoir? State your answer in kCh and gallons.

[10] Not really.

Exercise *1.20:* A medium-sized electric heater draws around 2 kW. The heater is on for three hours. How much energy has the heater used? State your answer in kWh and Joules.

Exercise *1.21:* The average US home uses 909 kWh of electricity in a month. Estimate how much this is in kWh per person per day. Assume that the average house has three people living in it.

Exercise *1.22:* Very roughly, what size power plant would be needed to generate sufficient electricity for a town 50,000 in Texas? Consider only the residential electricity needs of the town, not any electricity needed for commerce or industry. Be sure to explain how you arrived at your answer. The average house in Texas uses 1130 kWh or electricity each month.

Exercise *1.23:* The average home in Mexico uses 150 kWh of electricity each month.

1. Express this power in units of kW.
2. A 2 MW power station could provide approximately how many Mexican homes with electricity?

Exercise *1.24:* How long could you afford to keep a 50 W light bulb on in Maine if you had 10 dollars?

2
Charge and Current

2.1 Charge

Charge is a fundamental property of matter. Electrons have a negative charge and protons a positive charge.[1] Usually macroscopic objects have an equal number of electrons and protons, and so are electrically neutral. But sometimes an object might acquire a surplus (or deficit) of electrons, and thus would be negative (or positively) charged. Opposite charges attract; two differently charged objects exert forces on each other that tend to push those two object apart. Clever people can make use of this fact to make the electric force do useful things, like pumping water from a well, powering a car, or making an LED glow.

[1] I'm worried that this chapter is a bit too physics-ey. Can it be streamlined? Maybe. I think I can emphasize coulombs a lot less. But I'm gonna leave this be for now.

Electrons carry one fundamental unit of charge, usually denoted e and refereed to as the elementary charge. All electrons have a charge of $-e$, never a fraction of an e—always exactly e. And all protons have a charge of $+e$. Measuring charge in fundamental units—i.e., in units e—is useful when thinking about the chemistry of physics of single atoms or molecules, but not so useful when working with macroscopic phenomenon, which typically involve vast numbers of electrons. So an more convenient unit of charge is needed.

[2] In the official SI system of units, all unit names are rendered in lowercase, even those named after people. Abbreviations for units are capitalized if the unit derives from a proper name de la Convention du Mètre (2006), as is the case with coulombs, which are named after Charles-Augustin de Coulomb. We don't really like this capitalization convention, but we're not up for picking a fight with the Organisation Intergouvernementale de la Convention du Mètre, the international organization that oversees unit definitions and naming conventions. Move this note to Chapter 1 when Joules are first referenced?

That unit is the coulomb, abbreviated C.[2] It take a lot of elementary charges e to make up one coulomb. There are around 6.25×10^{18}, or 6.25 quintillion, electrons in one C. That's a lot. This means that the charge e, measured in C, of a single electron is very small:

$$1e = -1.60 \times 10^{-19} \text{coulombs} . \tag{2.1}$$

We will use coulombs as our unit of charge in the rest of the book. This is the standard SI unit, and while it is not ideal, it is much more convenient than measuring charge in terms of individual electrons. The standard symbol electric charge is Q.

2.2 Current

An electric current—or, more colloquially, electricity—is moving charge. The rate at which charge flows is known as *current* and is usually denoted by I.[3] The basic unit of current is the *ampere*, usually simply called an *amp* and abbreviated A. An amp is a current in which one coulomb flows every second:

$$1\text{A} = 1\text{C}/s\,. \tag{2.2}$$

[3] You may wonder why the letter I is used, since "I" is conspicuously absent from the word "current" and plays a supporting role in the word "electricity." The symbol I was used by André-marie Ampère who referred to electric current as *intensité de courant*—current intensity.

An Amp is a rate. Rates tell us how fast something happens. In this case, the thing happening is charges are moving. If something happens at rate r for a time interval t, the amount of thing that has happened is rt.[4] Thus, the amount of charge that has flowed if a current I flows for a time t is given by:

$$Q = It\,. \tag{2.3}$$

[4] Probably the most familiar application of this idea is distance, and you have likely heard the phrase "distance is rate times time." For example, if you are driving at 60 mi/hr for 2 hours, you will have traveled $60 \times 2 = 120$ miles.

The two examples below show several ways to use this equation.

Example 2.1. *A current of* 3 *mA (milliamps) flows through a wire into a capacitor for* 10 *minutes. How much charge will have accumulated in the capacitor? A capacitor can be thought of in this context simply as a bucket of charge. The idea is that charge flows through the wire into the bucket (capacitor), and we want to know how much charge has accumulated the bucket in* 10 *min.*

We are looking for Q, the amount of charge that has accumulated in $t = 10$ minutes. The relationship we want is $Q = It$, Eq. (2.3). So

$$Q = (3\text{mA})(10\text{min})\,. \tag{2.4}$$

One milliamp is 0.001 amps, and one amps is a coulomb per second. So

$$Q = (3\text{mA})\left(\frac{0.001\text{A}}{1\text{mA}}\right)\left(\frac{1\text{C/s}}{1\text{A}}\right)(10\,\text{min})\,. \tag{2.5}$$

Note that the A and mA units cancel, leaving us with:

$$Q = \left(0.003\,\frac{\text{C}}{\text{s}}\right)(10\,\text{min})\,. \tag{2.6}$$

We can't just multiply 3 and 10, since the units aren't right; we need to convert minutes into seconds. There are 60 seconds in a minute, so

$$Q = \left(0.003\,\frac{\text{C}}{\text{s}}\right)\left(\frac{60\text{s}}{1\text{min}}\right)(10\,\text{min})\,. \tag{2.7}$$

Now the units cancel. We multiply out and arrive at our final answer: The charge Q that has accumulated is 1.8 coulombs. By the way, if writing all these steps out seems like overkill, please have a look at our comments in Sec. B.1.

Example 2.2. *Suppose you need to have* 0.4 *C of charge flow through a wire in five minutes. What current would achieve this goal?*
We are looking to solve for the current I. Solving Eq. (2.3) for I, we obtain:

$$I = \frac{Q}{t} \, . \tag{2.8}$$

Plugging in, we have

$$I = \frac{0.4\,\mathrm{C}}{5\,\mathrm{min}} \, . \tag{2.9}$$

There are 60 seconds in one minute, so

$$I = \left(\frac{0.4\,\mathrm{C}}{5\,\mathrm{min}}\right)\left(\frac{1}{\frac{60\,\mathrm{s}}{1\,\mathrm{min}}}\right) \, . \tag{2.10}$$

Simplifying, we obtain

$$I = \left(\frac{0.4\,\mathrm{C}}{300\,\mathrm{s}}\right) = 0.0013\,\mathrm{C/s} = 0.0013\,\mathrm{A} \tag{2.11}$$

If we wanted, we could convert this to milliamps:

$$I = 0.0013\,\mathrm{A}\left(\frac{1000\,\mathrm{mA}}{1\,\mathrm{A}}\right) = 1.3\,\mathrm{mA} \, . \tag{2.12}$$

We will rarely work directly with coulombs, but we will work directly with amps. So it is good to have a feel for what an amp is and what some typical currents are in electrical situations you might be familiar with. Below is a list of approximate currents for a handful of applications.[5,6]

- Current flowing into the pump for a fishtank: 30 mA
- Current flowing into your smart phone when you charge it: 0.02 – 0.05 A
- Current flowing into a 60 W light bulb: 0.5 A
- Current flowing into a cable TV: 2 A[7]
- Current flowing into a hair dryer: 8 A
- Current flowing into a 1500 W electric heater: 12.5 A
- Current flowing into an electric dryer: 17 A
- Current flowing into a hot water heater: 25 A

One final note on current. The current at a point in a wire tells us how much charge is moving past that point each second. But this doesn't tell us how fast the charge is moving. One could get a particular current by having a small amount of charge moving quickly, or a large

[5] Should I format this as a table? Probably. But tables are a pain so let's keep it as a list for the time being.

[6] Also, I don't like the list formatting. I think there should be a bit less space between lines and the list should be indented. I can mess around with this later.

[7] why does it matter if it is a cable TV?

amount of charge moving slowly. An analogy with water in a river may help to make this clearer. If you know the flow rate—i.e., the current—in a river, you know how much water is moving past you each second. It could be the case that the river is narrow and water is moving very quickly. Or the river could be wide and deep, but moving slowly. Both could give rise to the same current.[8,9]

[8] Do we need this paragraph?

[9] At some point we should probably talk—as briefly as possible because it is confusing and doesn't really matter—about how the direction of current is the direction of positive flow and in reality in almost all materials it is negative electrons that move, and thus a current flowing to the right would actually consist of electrons flowing to the left.

2.3 *Voltage*

So far we have talked about current without talking about what makes it go. Why do electrons move down a wire? What pushes them along? The answer is voltage, also known as electric potential. Just like water flows from higher altitude regions to lower ones, charge tends to move from regions of higher potential to lower potential. What matters is the *difference* in potential. The greater the difference in potential from one end of a wire to another, the larger the current will be.

So what *is* voltage? This question turns out to be surprisingly abstract and probably not essential for understanding sustainable energy. It is pretty easy to get a handle on what gravity *does*: it makes things fall down. But what gravity really is, is a much more difficult and abstract question, and knowing the answer to this question doesn't help one stand up or build bridges or airplanes. Similarly, it is fairly easy to say what voltage does—it is something that makes current flow. But it's not easy to say what voltage really is. So we're not going to get too deep into what voltage is and instead will focus on what voltage does.[10]

[10] We will have a bit more to say about what voltage is in Sec. 3.1.

Dave likes to think of voltage as *oomph*. This is a made-up term that conveys the idea that voltage is something that pushes charge. Anna, being a engineer and not a theoretical physicist, doesn't always like to think in terms of oomph, but she tolerates it. Voltage is measured in units of *volts*. In Sec. 3.1 we'll talk about how volts are related to other, more familiar units.[11] It is tempting to think of voltage as a force, but this is not quite correct, as we'll see later on. But viewing voltage as a force gives the right idea.

[11] We can't properly do this now, because volts are related to energy, and we haven't introduced energy yet. But, at the risk of spoiling some of the fun from Chapter 3, one volt is equal to one joule per coulomb.

2.4 *Resistance and Ohm's Law*

So let's just say voltage is oomph. Given a certain amount of oomph, how much current will flow? To answer this question we need to introduce one more physical quantity: *resistance*. Resistance is what it sounds like—it is a property of a material that determines how it resists having charged oomphed through it. The greater the resistance, the less current will flow for a given amount of oomph. Or, the larger the resistance, the more oomph is needed to push a given amount of current.

This relationship is captured in a pleasingly simple equation:

$$V = IR\,. \tag{2.13}$$

This important equation is known as *Ohm's Law*. In this equation V is voltage and R is resistance. The standard unit for resistance is the *Ohm*, abbreviated Ω[12] and defined as follows:

[12] This is the capital Greek letter Omega. Dave thinks it looks like a bow-legged cowboy.

$$1\,\text{ohm} = 1\,\text{V/A} = 1\,\Omega\,. \tag{2.14}$$

Thus, if an object had a resistance of one ohm, a voltage of one volt would be sufficient to push an current of one amp through it. We'll do a few examples with Ohm's law in a moment. First, though, a few words about the the difference between current I and voltage V.

Current is, well, a current: it is moving charge. By analogy, we can picture water that is flowing in a pipe, as in Fig. 2.1. Along the pipe, there are no inlets or outlets, so the current must be the same everywhere. If it wasn't, water would be accumulating somewhere. That is, if at a certain point in the pipe the current flowing in was larger than the current flowing out, then the amount of water have to be increasing at that point, something that isn't possible in a closed, filled pipe. The picture is the same with electric current. We say that current flows through the wire, and the current is the same everywhere in the wire.

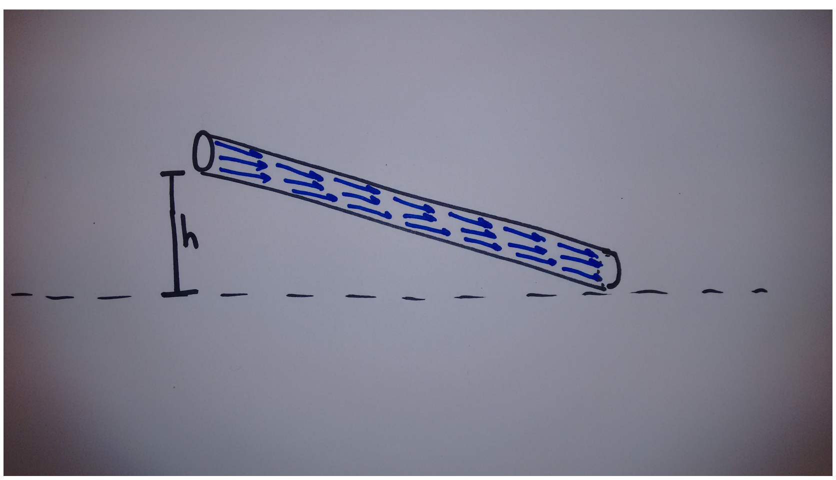

Figure 2.1: Water flowing through a tilted pipe. The current, or rate of flow, is the same everywhere in the pipe. The height h is roughly analogous to the "oomph", or voltage V.

The voltage, or oomph, can be thought of as being like the height difference between one end of the pipe and the the other. We would say that the altitude drops from one end to the other. Similarly, we would say that there is a voltage difference from one side of the wire to the other. One also refers to this as a potential difference. The main point is that the current is the same everywhere in wire or simple circuit without branches, while voltage drops from one side of the wire to the other.

2.5 *Ohm's Law Applications*

In this section we'll work through a few examples so you can see Ohm's law in action.

Example 2.3. *A certain light bulb has a resistance of* 10 *ohms. It is hooked up to* 1.5 *volt battery. What current flows through the light bulb?*

We start with ohm's law:

$$V = IR\,, \tag{2.15}$$

and solve for the current I:

$$I = V/R\,. \tag{2.16}$$

Plugging in, we obtain:

$$I = \frac{1.5\,\mathrm{V}}{10\,\Omega}\,. \tag{2.17}$$

To make sense of the units, recall that one ohm is one volt per amp. Thus,

$$I = \frac{1.5\,\mathrm{V}}{10\,\frac{\mathrm{V}}{\mathrm{A}}} = 0.15\,\mathrm{A}\,. \tag{2.18}$$

2.6 *Batteries and Amp·hours**

* This chapter can safely be skipped. It is only needed for Chapter N on storage. Or will it be needed at all?

A battery is a device that maintains a voltage difference across its two terminals. If a wire is attached to be battery, connecting one terminal to the other, then a current will flow through the wire. How much current depends on the voltage of the battery and the resistance of the wire. Different batteries have different voltages. For example, in Fig. 2.2 is a familiar AAA battery. The potential difference between the two terminals is 1.5 volts.

Figure 2.2: The AAA battery that Dave used to power his noise-canceling headphones.

Almost all commonly used round batteries such as AA, AAA, and D batteries are 1.5 volts. The rectangular battery whose terminals are "snaps" are 9-volt batteries. An example is shown in Fig. 2.3. The batteries in most conventional cars are 12-volt batteries.

Sadly, as you have no doubt experienced, batteries do not last forever. As noted above, a battery is a device that maintains a voltage difference. If you hook the battery up to something, current flows. This current is moving charge—and the charge comes from the battery. Eventually, the battery runs out of charge. So an important characteristic of a battery, in addition to the voltage it maintains, is how much charge can flow out of it before it poops out. This amount of charge is known as the battery's *capacity*.

Figure 2.3: A 9 volt battery. (Image by Lead holder (Own work) [CC BY-SA 3.0 (http://creativecommons.org/licenses/by-sa/3.0) or GFDL (http://www.gnu.org/copyleft/fdl.html)], via Wikimedia Commons.)

Capacity is an amount of charge. As discussed in Sec. 2.1, the standard SI unit for charge is the coulomb. So one would expect that one would measure capacity in coulombs. This is certainly reasonable and logical, but actually another unit is used instead: the *amp-hour*. An amp-hour is the amount of charge that has flowed through a circuit if a current of an amp flows for one hour. Let's figure out how to convert from amp-hours to coulombs.

Example 2.4. *How many coulombs are in one amp-hour?*
We will start with the relationship $Q = It$ and recall that one amp is one coulomb per second:

$$Q = It = 1\,\mathrm{A} \times 1\,\mathrm{h} = \left(1\frac{\mathrm{C}}{\mathrm{s}}\right) \times 1\,\mathrm{h}\,. \tag{2.19}$$

Converting hours into seconds, we obtain

$$Q = \left(1\frac{\mathrm{C}}{\mathrm{s}}\right)(1\,\mathrm{h})\left(\frac{60\,\mathrm{min}}{1\,\mathrm{h}}\right)\left(\frac{60\,\mathrm{s}}{1\,\mathrm{min}}\right) \tag{2.20}$$

Multiplying out, we find that one 1 amp-hour $= 3,600$ coulombs.

Amp-hours are perhaps a bit clumsy, but they are the standard unit for battery capacity, so we need to get comfortable with them. To that end, let's do one more example.

2.7 *Exercises*

Exercise *2.1:* A piece of plastic happens to have one billion more electrons than protons. What is the net charge, in coulombs, of the piece of plastic?

Exercise *2.2:* The current flowing through a wire is 30 mA.

1. How much charge flows through the wire in 5 minutes?
2. How long must the current flow for 100 coulombs to have flowed through the wire?

Exercise *2.3:* About how many coulombs does it take to charge your phone?

Exercise *2.4:* When charging your phone, how many electrons per second are flowing through your charger into the phone?

Exercise *2.5:* Find the current flow through a light bulb from a steady movement of

1. 60 Coulombs in 4 seconds
2. 15 Coulombs in 2 minutes
3. 3×10^{22} Coulombs in 1 hour.

Exercise *2.6:* Will a current of 25,000 coulombs per hour cause a 5 amp fuse to blow?[13]

[13] A five amp fuse would blow of five or more amps of current flow through it.

Exercise *2.7:* How many coulombs are 200 mAh?

Exercise *2.8:* An average car battery has a capacity of approximately 48 Ah. Headlights draw about 10 A. How long will it take a fully charged battery to discharge completely if the headlights are left on after the car is turned off?

Exercise *2.9:* How long can a 4.5Ah, 1.5V flashlight battery deliver 100mA?

Exercise *2.10:* What voltage is needed to push a 0.5 amp current through a 100 ohm load?

Exercise *2.11:* A current of 3 amps flows for 20 minutes? How many coulombs is this?

Exercise *2.12:* What voltage is needed to push 5 amps through a 20 Ω load?

Exercise *2.13:* A current of 5 amps is pushed by a voltage of 60 volts through a light bulb. What is the resistance of the bulb?

Exercise *2.14:* A current of 0.5 amps flows for 2 hours. How many amp-hours is this? How many coulombs?

Exercise *2.15:* How long can a 12 volts, 30 Ah battery deliver a current of 0.5 amps?

Exercise *2.16:* You need a 1.5 volt battery to deliver light up a 5 ohm light bulb for 50 hours. What should be the capacity of the battery?

Exercise *2.17:* You have a battery that has sufficient capacity to light up a 10 ohm light bulb for 30 hours. Suppose that this battery's capacity was doubled, but all other properties of the battery remained the same.

1. For how long could this new battery light up the bulb?
2. What current would the new battery push through the light bulb?
3. What voltage does the new battery deliver?

Exercise *2.18:* A voltage of 120 volts is applied across a heater that has a resistance of 9.6 ohms. How much current flows through the resistor?

Exercise *2.19:* A small 1.55 v battery a capacity of 200 mAh.[14] Suppose you use this battery to power a microphone that has a resistance of 775 ohms.

1. What current flows through the microphone?
2. How long could you power the microphone with this battery?

[14] Dave uses this battery to power a small external microphone that he uses when recording math videos. See http://www.amazon.com/Enercell-200mAh-Button-Battery-3-Pack/dp/B007TV7V0U.

ïż£

3
Energy in Circuits

3.1 Energy in Circuits

In Chapter 2, we referred to voltage as *oomph*. Voltage is what pushes current through, say, a light bulb. The greater the voltage across the light bulb, the greater the current. Now that we have introduced the notion of energy, we can say more directly what voltage is: *A voltage of one volt would give one coulomb of charge one joule of energy*. That is

$$1\,\text{volt} = \frac{1\,\text{joule}}{1\,\text{coulomb}} \,. \tag{3.1}$$

This means that if one coulomb were pushed by one volt, the charge would have gained one joule of energy. In general, a voltage is physical quantity measuring "oomphiness". Voltage measures energy per charge.

We can use this fact to come up with a formula for the power associated with an electric current. The power P associated with a current I pushed by a voltage V is given by:

$$P = VI \,. \tag{3.2}$$

It may be helpful to think about the units in this equation:

$$P = VI = \left(\frac{\text{J}}{\text{C}}\right)\left(\frac{\text{C}}{\text{s}}\right) = \text{J}/\,\text{s} = W \,. \tag{3.3}$$

So the formula ends up giving up power in watts, as one would expect.

In Chapter 1 we mentioned that energy comes in different forms, each with its own formula. And we presented such formula: namely, the formula for kinetic energy: $E = (1/2)mv^2$. Equation (3.2) gives us another formula. When considering electricity, one works with power, not energy. Electricity is a flow of current, so it is natural to think in terms of the flow of energy—i.e., power.

Example 3.1. *In the U.S. the voltage of a normal home outlet is* 120 *V. If a lamp with a* 50 *W light bulb is plugged in to this outlet, what current flows through the bulb?*
We need to solve Eq. (3.2) for I. Doing so, we have

$$I = P/V \,. \tag{3.4}$$

Plugging in the values given for P and V, we obtain

$$I = 50\,\mathrm{W}/120\,\mathrm{V} = \frac{50\,\mathrm{J/s}}{120\,\mathrm{J/C}} = 0.41\,\mathrm{C/s} = 0.41\,\mathrm{A} \,. \tag{3.5}$$

3.2 *The Cost of Electricity*

Electric utility companies are responsible for providing your house with electricity. You then use this electricity to make toast, run your refrigerator, light up your house when it is dark outside, and so on. This electricity isn't free, however. The utility company charges you for every kWh or electricity you use. Here in Maine, the cost is 0.17 for on kWh.[1] The average household in Maine uses 520 kWh of electricity in a month. This would cost:

[1] Include a table with costs for other states. Would also be of interest to list power costs in a few other countries.

$$520\,\mathrm{kWh}\left(\frac{\$0.17}{1\,\mathrm{kWh}}\right) = \$88.40 \,. \tag{3.6}$$

Household electricity use in Maine is well below the national average. The primary reason for this is mostly likely that in most houses in Maine there is no need for air conditioning owing to Maine's coolish climate. Air conditioning is a significant use of residential electricity.[2]

[2] Include a few problems about this?

Figure 3.1 shows one of Dave's utility bills. The local utility where we live is a company called Emera. We see that the period of the bill is around one month: from July 15 to August 16, 2014. The total energy Dave's house used during this time period is 215 kWh. This is well below the average for Maine. There are several reasons why Dave uses so little electricity. First, electric hot water heaters tend to be the largest use of home electricity. However, Dave does not have an electric hot water heater; he has one that runs on gas. His clothes drier also runs on gas. He probably didn't use it much, however, since in the summer he does most of his drying on a clothesline in his backyard. The second reason Dave's utility bill is so small is that only one other person lives in his house, his wife. Dave and his spouse have no kids.

So Dave used 216 kWh of electric energy during this 32-day period and was charged $38.48. This charge is broken into two parts, supply and delivery. The supply part of the bill is the charge associated with generating the electricity on Fig. 3.1 toward the bottom, we see that the

Emera Maine Customer Service Center
207-973-2000
1-855-Emera11(363-7211)
Email: custserv@emeramaine.com
Pay Online at www.emeramaine.com

Account Information	
Account Number:	[redacted]
Service Address:	DAVID P. FELDMAN [redacted]
Read Cycle:	14M
Next Planned Meter Read:	13-SEP-2014

Customer Account Summary

Previous Statement Balance	Payments (-) Thank You	Adjustments (+/-)	Balance Before New Charges	New Charges	Current Account Balance	Total Now Due
[redacted]	[redacted]	$0.00	$0.00	$38.48	$38.48	$38.48

Energy Comparison

	This Month	Last Month	One Year Ago
KWH	216	211	256
Service Days	32	30	32
KWH Per Day	6.8	7.0	8.0
Cost Per Day	1.20	1.25	1.28

Residential Service (Service 1)

Meter Number	Units	For Service From	To	Days	Meter Reading Current	Previous	Constant	KWH
007160840	1	07/15/14	08/16/14	32	27701	27485	1	216

Message from Emera Maine

Emera Maine Delivery (Service 1 Rate Code A000)

Distribution Energy	216 kWh @ 0.0762400	$16.47
Transmission	216 kWh @ 0.0261800	$5.65
Balance Forward		**$0.00**
Total Emera Maine Delivery Charges Due		**$22.12**

Message from your Supplier

Your electricity price for Standard Offer service for the period of March 1, 2014 through February 28, 2015 is $0.0757596 per kWh.
For information on buying green power go to www.maine.gov/greenpower

Standard Offer Supply (Service 1 Rate Code 1000 Class S)

Electricity Supply	216 kWh @ 0.0757596	$16.36
Balance Forward		**$0.00**
Total Standard Offer Supply Charges Due		**$16.36**

Figure 3.1: Dave's electricity bill from the summer of 2014.

supply cost per kWh is $0.0757596. Multiplying this cost by the 216 that Dave used, one obtains $16.36.

There is also a charge associated with the delivery service provided by Emera—actually getting the electricity from the power plant to your house. The delivery service is broken down into two parts, *distribution energy* and *transmission*. The rate for distribution energy is 0.762400 per kWh and the rate for transmission is 0.0261800 per kWh. Multiplying these rate by the 216 kWh that Dave used yields $22.12. The total bill for this month is the sum of the delivery and supply charges.

The difference between transmission and distribution is not an important one, but here are a few words on it, anyway. Transmission refers to the transmission of power from the generator two a distribution center. Transmission is often "long-haul"—large power lines that go large distances and perhaps cross state boundaries. Distribution refers to moving the power from a distribution center out through neighborhoods and into homes. I wonder if this discussion shouldn't go in the chapter on the grid.

Add paragraph about how Dave could choose to buy something other than the standard offer electricity. This would mean the energy is generated not by Emera, but Emera's power lines would still deliver the electricity. For example, to buy power from the company Maine Green Power would cost an additional $0.015, or 1.5 cents, per kWh.[3] Maine Green Power generates their energy from a mix of renewable sources in Maine, including hydro, solar, wind, and biomass.

[3] Actually, it's a bit more complicated than this, since Maine Green Power charges not by the kWh but in bundles of 500 or 250 kWh per month. See https://megreenpower.com/overview/faq.

3.3 Demand Pricing

Electricity use is not constant throughout the day. This poses challenges to utility companies, who by law are required to meet electricity demand of consumers. If lots of people return home after work around 6:00 PM, and everybody wants to turn on their air conditioners and plug in the electric cars they just drove home, the utility company has to meet that demand. It cannot make the customer wait. This surge in electricity poses a challenge to the utility company. It needs to have power in reserve to meet a sudden spike in demand. Doing so is difficult from engineering perspective and is costly.

So the utility company would like to even out demand—have less people use electricity during the peak times of 5:00–7:00pm and more people use electricity during low-demand times late at night. To do so, many utility companies offer different prices for electricity depending on the time of day. Electricity is more expensive in the late afternoon and is cheaper late at night.

An example of this is the tiered base plan offered by PG&E (Pacific Gas and Electric Company) in northern California. For more, see: http://www.pge.com/en/myhome/myaccount/explanationofbill/etoub/index.page. We should likely say a bit more about this and work through an example or two.

3.4 Residential vs. Commercial Meters

Maine takes a different approach to pricing electricity. There are three different rate structures, referred to as different meter types.

3.4.1 Class A

This is the type of meter for homes with low or average electricity demand. Dave's home, and almost all homes in Maine, are on a type A meter. The rates are:

1. Supply Energy: 0.0757596$/kWh

2. Distribution Energy: 0.0762400 $/kWh

3. Transmission Energy: 0.0261800 $/kWh

So the total rate is: 0.1781796 $/kWh.

3.4.2 Class B

These meters are for small commercial buildings or homes with a large electricity demand. Customers are billed based on how much energy they use, and are also charged an additional monthly fee. The rates are

1. Supply Energy: 0.07720$/kWh

2. Distribution Energy: 0.05089 $/kWh

3. Transmission Energy: 0.02512 $/kWh

So the total rate is: 0.15021 $/kWh

The monthly fee is $13.08.

Note: The info for Class B and Class C meters is a bit over a year old. For your projects, you'll want to get updated rate information from Emera. See `http://www.emeramaine.com/business/rates/rates-schedules/`.

3.4.3 Class C

Class C meters are for medium-sized commercial buildings with a high electricity demand and whose peak power demand is at least 25 kW. Recall that power companies don't like big surges in demand. They would much rather you use energy at a steady rate rather than use a lot of it all at once. The reason for this is that large peak demands require additional power capacity in the grid, which is expensive.

So class C meters have a low per kWh charge and a high penalty for peak power. This provides business-owners with a strong incentive to even out their energy consumption and avoid large peak demand. The details of this type of meter are:

1. Distribution Demand: $9.04/kW

2. Transmission Demand: $8.38/kW

3. Distribution Energy: $0.00896/kWh

4. Supply Energy: $0.07420/kWh

Note that the demand charges are in units of power (kW), whereas the energy charges are in units of energy (kWh).

So the total energy charge is 0.08316 $/kWh and the total demand (power) charge is $17.42/kW. In addition, there is a monthly charge of $40.27.

3.5 Thinking Globally

Remind readers that residential electrical energy use is a small fraction of overall energy use.

Somewhere in this chapter mention feed-in tarrifs and refer readers to Sec. 19.3 for more details.

Do we want this section? At some point we need to connect up with the larger picture. Where does this belong?

3.6 Exercises

Exercise *3.1:* A toaster draws 1000 W from a household socket in the U.S. What current is flowing through the toaster?

Exercise *3.2:* Two amps flow out of a 220 V outlet to run an electric dryer. What power is being used by the dryer? If the dryer runs for 3 hours, how much energy has been used? State your answer in kWh.

Exercise *3.3:* If Dave buys an electric heater for his home office and leaves it on for 5 hours per day, 5 days a week for the month of January. How much will it add to his electricity bill assuming he is on an A meter and the heater draws 1kW?

Exercise *3.4:* The March 2013 electricity bill for Turrets as shown here: `https://sites.google.com/a/coa.edu/college-of-the-atlantic-archive-of-sustanable-energy-projects/home/class-c000-meters` shows charges of $836.13 from Bangor Hydro (including balance forward) for Transmission and Distribution and $786.52 from Constellation Energy for supply (commonly referred to as generation) bringing the total monthly expense to $1622.65.

1. What would the difference in cost be if Turrets were an A meter rather than a C meter?
2. In southern Maine CMP has increased C-meter energy rates to $.12/kwh (leaving peak power demand rates at current level). What would be the impact on this total if the same rate hike were levied against this bill?
3. What would have offered more savings: (1) if energy consumption had been reduced by 50% or (2) if peak power demand had been reduced by 50%?

Exercise *3.5:* An Emera customer discovered that her meter has been incorrectly classified as a B meter rather than an A meter for the past year. If she used 622 kWh per month for 12 months was she over or under charged by the company and by how much

Exercise *3.6:* A business uses 2000 kWh in a month and has a peak draw of 30 kW.

1. What is the business's power bill if it is on a C meter?
2. If the business reduced its peak draw and so was reclassified to a B meter (but still used 2000 kWh of energy), what would its power bill be?

Exercise *3.7:* A business uses 20000 kWh in a month and has a peak draw of 30 kW.

1. What is the business's power bill if it is on a C meter?
2. If the business reduced its peak draw and so was reclassified to a B meter (but still used 20000 kWh of energy) what would its power bill be?

4
Generating Electricity

In this chapter we'll talk about generating electricity. One could, we suppose, do sustainable energy without the material in this chapter. But electricity is so fundamental to the world that I think knowing a bit about how electricity is "made" is pretty important. So in this chapter we'll go over the basic physical phenomenon that lies behind almost all modes of electric power generation.

4.1 Electromagnetic Induction

more blah blah.

4.2 Different Ways of Turning Turbines

4.3 Nameplate Capacity and Capacity Factor

The amount of power that can be delivered by a generator under ideal conditions or is known as the *nameplate capacity* of the generator. For a gas or coal plant, the nameplate capacity is the power generated when it is running at "full blast"—the maximum output. For a set of solar panels, the nameplate capactiy would be the power produced on a sunny afternoon.

Generators do not operate at nameplate capacity all the time. For example, a wind turbine might have a nameplate capacity of 2MW. This means that under ideal windy conditions the turbine will generate 2MW of power. But conditions aren't always ideal. So often the turbine is producing less than 2MW. The *capacity factor* measures the actual average power production, expressed as a percentage of the maximum value. An example will make this clearer.

Example 4.1. *A 2MW wind turbine has a capacity factor of* 0.3*. How much energy does it generate in one year?*

If the turbine produced 2MW for one year, the amount of energy produced would be:

$$\text{Energy} = \text{Power} \times \text{Time} = 2\,\text{MW} \times 1\,\text{yr}\left(\frac{365\,\text{d}}{1\,\text{yr}}\right)\left(\frac{24\,\text{h}}{1\,\text{d}}\right) = 17,500\,\text{MWh} = 17.5\,\text{GWh}\,. \tag{4.1}$$

We now account for the capacity factor:

$$\text{Energy Produced} = \text{Capacity Factor} \times 17.5\,\text{GWh} = (0.3)(17.5\,\text{GWh}) = 5.25\,\text{GWh}\,. \tag{4.2}$$

One could, of course, combine the above two equations and do this problem in one step intead of two.

Capacity factors are usually calculated over a one-year period. That is, one looks at the actual energy generated and compares it to how much energy would have been produced if the generator operated at its nameplace capacity for a year. This is illustrated in the next example.

Example 4.2. *The Hoover Dam in the Southwestern US has a nameplate capacity of 2080 MW. The average annual energy generated by the powerplant from 1947 to 2008 was 4.2 billion kWh. (Data source:* `https://www.usbr.gov/lc/hooverdam/faqs/powerfaq.html`*, accessed September 12, 2017.) What is the capacity factor for the Hoover dam for this time period?*
Let's first figure out what the maximum amount of energy the Hoover Dam could produce in a year. This correspondes to one year of producing at a rate of 2080 MW, or 2.08 GW.

$$\text{Energy} = \text{Power} \times \text{Time} = 2.08\,\text{GW} \times 1\,\text{yr}\left(\frac{365\,\text{d}}{1\,\text{yr}}\right)\left(\frac{24\,\text{h}}{1\,\text{d}}\right) = 18220\,\text{GWh} = 18.2\,\text{TWh}\,. \tag{4.3}$$

The actual amount of energy produced in an average year is 4.2 billion kWh. Let's convert kWh to TWh. Recalling that Tera $= 10^{12}$, and one billion $= 10^9$:

$$4.2 \times 10^9\,\text{kWh}\left(\frac{1000\,\text{Wh}}{1\,\text{kWh}}\right)\left(\frac{1\,\text{TWh}}{10^{12}\,\text{Wh}}\right) = 4.2\,\text{TWh}\,. \tag{4.4}$$

We can now calculate the capacity factor:

$$\text{Capacity Factor} = \frac{\text{Actual Production}}{\text{Max Production}} = \frac{4.2\,\text{TWh}}{18.2\,\text{TWh}} = 0.23\,. \tag{4.5}$$

Note that the capacity factor is dimensionless; being a ratio it does not carry any units.

4.4 *Exercises*

Exercise *4.1:* The nameplace capacity of the Three Gorges Dam in China is 22, 500 MW. In 2015 it generates 87 TWh of energy. What is its capactiy factor?

ïż£

5 Thermal Energy

5.1 Heat

In this chapter we explore another type of energy: heat. What we call "heat" is actually hidden kinetic energy—the motion of the molecules that make up an object. For this reason "heat" is usually referred to in physics as internal energy. (For physicists, heat has a slightly restricted meaning: heat refers to any energy that flows from one object to another as a result of a temperature difference between those two object. So, for example, heat flows from a warm woodstove into the air. In so doing, the internal energy of the woodstove decreases and the internal energy of the air increases.) Since this book isn't written for physicists, I will not be militant about the physics meaning of the word heat and will sometimes refer to internal energy (improperly) as heat.[1]

[1] Add citation(s) to physics-ey discussion of heat vs. energy? Perhaps Moore or Schroeder?

We have an equation for kinetic energy, $E = (1/2)mv^2$, and for the power in an electric current, $P = VI$. We need an equation for heat. Here it is:

$$\Delta U = mc\Delta T \,. \tag{5.1}$$

In this equation ΔU is the change in internal energy of an object and m is the object's mass. The quantity ΔT is the change in object's temperature, and c is the object's specific heat. We don't want to get too deep into thermal physics, but a brief discussion of temperature, internal energy, and specific heat is in order.

Internal energy is the amount of hidden kinetic energy in an object. In a solid, the molecules are bound chemically to other molecules, but molecules can vibrate. The kinetic energy of this motion contributes to the internal energy of the solid. In a gas or liquid, molecules are free to move around, and molecules also can rotate and vibrate. The details of all of this can get a bit involved and is typically studied at some length in a course on thermal physics or statistical mechanics. For our purposes, the key picture is that internal energy refers to the kinetic energy associated with moving, wiggling, and vibrating molecules.

We refer to this kinetic energy as hidden, because you can't see it. If someone throws a rock at you, the kinetic energy is immediately apparent, because you can see the rock moving and you could calculate its kinetic energy[2] using $K = (1/2)mv^2$. The rock also has some internal energy: i.e., heat. But you can't see this, because molecules are far too small to be visible. But heat can be experienced: putting ones hands around a pleasantly warm cup of coffee or accidentally touching a hot pan while cooking.

[2] You should duck, first, so the rock doesn't hit you. Then calculate the energy.

Informally, temperature is how hot something it is. But what is temperature more formally? This question can be answered at a number of different levels, some of which are quite abstract and subtle. One way of thinking of temperature is that it is a property of an object that determines if heat will flow from or two it: heat flows spontaneously from hot objects to cold. Equivalently[3], temperature is a measure of the average energy per degree of freedom of an object. Degree of freedom in this context refers to a "place" where kinetic energy could be hidden. Examples of degrees of freedom are the various ways a molecule can vibrate or rotate, and (if not in a solid) the directions in which a molecule can move. Objects that are hot have, on average, a lot of energy per degree of freedom. When a hot object comes in contact with[4] a cold object, which does not have a lot of energy per degree of freedom, the hotter object loses energy while the colder one gains energy.

[3] It's not obvious that these two views of temperature are equivalent.

[4] I.e. exchanges energy with

Finally *specific heat* is a property of an object that tell us how much energy is needed to raise a kilogram of that object by one degree. If an object has very many degress of freedom per kilogram, then it will take a lot of energy to raise its temperature by one degree. Such an object has many places to hide kinetic energy. So it can absorb a lot of energy without having the average energy per degree of freedom increase much. Different materials have different specific heats. In almost all cases, specific heats are determined experimentally. They are a property of a material, like density or conductivity, that one looks up in a book.

In this book the materials we will be concerned with are air and water. Their specific heats are:

- Specific Heat of Water ≈ 4200 J/(kg·K)
- Specific Heat of Air ≈ 1200 J/(m^3·K)

Since a liter of water has a mass of one kilogram, the specific heat of water is often expressed in units of Joules per liter per Kelvin. Note that the specific heat for air is given per volume (m^3) and not per mass.

Example 5.1. *How much energy is required to heat* 4 *kg of water from* 20 *to* 100 *degrees Celsius? Answer in both Joules, kWh, and BTUs.*

Using Eq. (5.1), we have

$$\Delta U = mc\Delta T = (4\,\mathrm{kg})(4200\,\mathrm{J/\,kg\,K})(80\,\mathrm{K}) \approx 1340000\,\mathrm{J} = 1.34\,\mathrm{MJ}\,. \tag{5.2}$$

Converting to kWh (recall that one kWh equals 3.6 mega Joules), we find:

$$1.34\,\mathrm{MJ} = 1.34\,\mathrm{MJ}\left(\frac{1\,\mathrm{kWh}}{3.6\,\mathrm{MJ}}\right) \approx 0.37\,\mathrm{kWh}\,. \tag{5.3}$$

We convert to BTU by using the fact that one kWh equals 3412 BTU;

$$0.37\,\mathrm{kWh} = 0.37\,\mathrm{kWh}\left(\frac{3412\,\mathrm{BTU}}{1\,\mathrm{kWh}}\right) \approx 1270\,\mathrm{BTU}\,. \tag{5.4}$$

Example 5.2. *Consider a house that is 2000 square feet. Assume that the ceilings are 8 ft. Let's say that it is 0 degrees C outside and you want the air inside your house to be 20 degrees. Estimate how much energy it would take to raise the air in this house 20 degrees C. Express your answer in Joules and kWh. If the air in this house changes every two hours, what is the daily energy needed for heat? How much would this cost in Maine if this heat was electric?*
The volume of air in the house is (8ft)(2000ft^2) = $16,000$ ft^3. Let's convert this volume to cubic meters. There are 3.3 feet in one meter. So

$$16,000\,\mathrm{ft}^3 = 16,000\,\mathrm{ft}^3\left(\frac{1\,\mathrm{m}}{3.3\,\mathrm{ft}}\right)^3 \approx 445\,\mathrm{m}^3\,. \tag{5.5}$$

Note that, since we are dealing with volumes, the conversion factor gets cubed. We can now use Eq. (5.1) to determine how much energy it would take to warm up this volume of air by 20 degrees:

$$\Delta U = mc\Delta T = (445\,\mathrm{m}^3)(1200J/(\,\mathrm{m}^3\,\mathrm{K}))(20\,\mathrm{K}) \approx 10,700,000\,\mathrm{J} = 10.7\times 10^6\,\mathrm{J} = 10.7\,\mathrm{MJ}\,. \tag{5.6}$$

So this is the energy needed to warm up the cold air that enters the house. We can now convert to kWh:

$$10.7\,\mathrm{MJ} = 10.7\,\mathrm{MJ}\left(\frac{1\,\mathrm{kWh}}{3.6\,\mathrm{MJ}}\right) \approx 3.0\,\mathrm{kWh}\,. \tag{5.7}$$

In a drafty house the air might turn over every two hours. What this means is that, in effect, every two hours all the warm air has left your house to be replaced by cold air from the outside. This air will need heating. We've seen that it takes around three kWh to heat up the air in your house once. If we need to do it twelve times a day, then heating the air in your house will take 36 kWh/day.

5.2 BTUs

Heat is energy. So one can measure it in Joules, which is the standard metric (SI) unit, or kWhs, which is the standard unit for measuring electrical energy and is a convenient unit for everyday, individual energy consumption.[5] It is common, at least in the US (and Canada?) to use other units for thermal energy. The most important of these are

[5] For example, the average American uses 250 kWh of energy in a day. This is equivalent to 900 million joules.

British Thermal Units, or BTUs. A BTU is the amount of energy needed to raise one pound of water by one degree Fahrenheit.

To be honest, I don't have anything nice to say about BTUs. One might think that then I shouldn't say anything at all, but this isn't an option in this instance, because BTUs are the standard unit for thermal energy. British Thermal Units are related to kWhs via the following conversion:

$$3412\ \text{BTU} = 1\,\text{kWh}\ . \tag{5.8}$$

So like joules, BTUs are annoying small.[6] In case you wanted to convert from BTU to Joules or vice versa, here's the relevant conversion factor:

[6] For example, 250kWh are 853,000 BTU.

$$1BTU = 1055J. \tag{5.9}$$

This seems like a good time remind you that conversion factors and lots of other useful facts are collected in Appendix D.

British Thermal Units are small, so it is common to work with thousand or millions of BTUs. Unfortunately, BTUs don't follow the typical kilo and mega conventions. Perversely, one thousand BTUs are denoted 1 MBTU. Presumably, this is because M denotes 1000 in Roman numerals.[7] What about one million BTUs? It would make sense for these to be abbreviated MBTU, but this is already taken by 1000 BTU so instead MMBTU is used. To summarize:

[7] At least this is what is claimed in an uncited remark on the wikipedia page for BTUs.

- 1 MBTU = 1000 BTU ,
- 1 MMBTU = 1,000,000 BTU .

I like to pronounce MMBTU as "mmmmmmmm B.T.U.", but this is not a standard pronunciation. Yet.

Finally, there is one other thing you should know about BTUs. I've saved the worst for last. A BTU is a unit of energy. However, it is also used as a unit of power. The power delivered by furnaces and heaters is very often described in terms of BTUs. For an example of this, see Fig. 5.1. When this is done, what it means is that the furnace is delivering a certain amount of BTUs (energy) per hour (time).

The fact that BTUs are used for both energy and power is really an awful thing—far more so than the fact that a million BTUs is an MMBTU. Power and energy are different physical quantities and so cannot possibly be represented by the same unit. However, the convention of using BTU to mean BTU/h isn't going away any time soon. Usually it is not difficult to tell from the context if one means BTU or BTU/h, so in practice there isn't likely to be much confusion, even if it is unnecessarily ambiguous (and just plain wrong).

Before moving on, there is one commonly used BTU-related unit that I need to mention. A *therm* is $100,000$ BTUs and is roughly the energy released if 100 cubic feet of natural gas are burned. One hundred cubic

Figure 5.1: A furnace for sale at Home Depot. Its power is listed in BTUs. What this means is that this furnace is capable of producing 60,000 BTUs per hour.

feet is sometimes abbreviated CCF. There are slight differences among the definitions of the therm in the US, the UK, and the EU. These differences are less than 0.1 percent, so one needn't worry about them. Depending on your location, natural gas is sold in units of CCF, therms, MJ, or kWh.

5.3 *Properties of Different Fuels*

5.3.1 *Calorific Value of Fuels*

As you are now doubt aware, when you light something on fire it burns and makes things hot. How much thermal energy do you get from this fire? That depends on what you're burning. Different fuels give off different amounts of energy per kg or per liter when burned. These quantities are known as a fuel's *calorific value*.

- Gasoline: 13.0 kWh/kg, 34.7 MJ/L, 120,480 BTU/gallon
- Coal: 8.0 kWh/kg, 19,100,000 BTU/short ton[8].
- Propane: 13.8 kWh/kg, 25.4 MJ/L, 91,600 BTU/gallon
- Natural gas: 14.85 kWh/kg, 0.04 MJ/L, 1,037 BTU/ft^3
- Heating oil: 12.8 kWh/kg, 37.3 MJ/L, 139,000 BTU/gallon
- Kerosene: 12.8 kWh/kg, 37 MJ/L, 135,000 BTU/gallon
- Wood: ~4–5 kWh/kg

[8] A short ton is 2000 pounds

- Hardwood: 24,000,000/cord
- Pine: 18,000,000/cord

The BTU data can be found at https://www.eia.gov/energyexplained/index.cfm?page=about_energy_units. I need a source for the hardwood and pine. Other data is from MacKay.

5.3.2 *Carbon Intensity of Fuels*

How much CO_2 is released into the atmosphere if you get a kWh worth of thermal energy? Again, it depends on what you're burning. The *carbon intensity* of a fuel is the grams of CO_2 released if you burn enough to produce 1 kWh of thermal energy. Some carbon intensities are listed below.

- Natural gas: 190
- Propane: 217
- Gasoline: 240
- Diesel: 250
- Fuel oil: 260
- Coal: 300

5.3.3 *Costs of Fuels*

Fuel costs are notoriously volatile and fluctuate considerably from year to year and even from month to month. They also vary widely in different parts of the world. Current average fuel prices for Maine can be found at http://www.maine.gov/energy/fuel_prices/. The US Energy Information Administration also tracks weekly fuel prices at: https://www.eia.gov/dnav/pet/pet_pri_wfr_dcus_nus_w.htm

At the present moment[9], average prices in Maine are:

[9] September 2017

- Natural gas: $1.31/therm
- Heating oil: $2.03/gallon
- Wood pellets: $261/ton
- Kerosene: $2.56/gallon
- Propane: $2.36/gallon
- Cord wood: $250/cord

Having just looked up this data, I'm a bit troubled by the fact that I'm currently paying $3.70/gallon for propane. This might be something to look into. The wood price, however, is what I've paid per cord for the last few years.

Example 5.3. *Suppose you burn* 20 *gallons of heating oil. How much energy is this, in kWh and BTUs? How much* CO_2*has been emitted into the atmosphere?*
I know the calorific value of heating oil in units of MJ/L. So I'll convert to liters first:

$$20\,\text{gal}\left(\frac{3.8\,\text{L}}{1\,\text{gal}}\right) = 76\,\text{L}\,. \tag{5.10}$$

Using the data listed above, we can figure out how much thermal energy we get from burning this amount of heating oil:

$$76\,\text{L}\left(\frac{37.3\,\text{MJ}}{1\,\text{L}}\right) = 2800\,\text{MJ}\,. \tag{5.11}$$

Let's now go to kWH:

$$2800\,\text{MJ}\left(\frac{1\,\text{kWh}}{3.6\,\text{MJ}}\right) = 780\,\text{kWh}\,. \tag{5.12}$$

And next to BTU:

$$780\,\text{kWh}\left(\frac{3412\,\text{BTU}}{1\,\text{kWh}}\right) = 2,660,000\,\text{BTU}\,, \tag{5.13}$$

which is 2.6 MMBTU, recalling that 1 MMBTU = 1,000,000 BTU.

The carbon intensities of fuels are given above in units of grams of CO_2per kWh of thermal energy. Since we calculated above that 780 kWh of thermal energy have been produced, we can find the CO_2that has been released:

$$780\,\text{kWh}\left(\frac{260\,\text{gal}}{1\,\text{kWh}}\right) = 203000\,\text{gal} = 203\,\text{kg}\,. \tag{5.14}$$

5.4 *Efficiencies*

When one burns something to heat ones home, not all of the thermal energy released by combustion goes into the air or water that you're trying to warm up. Some of it inevitably goes up the chimney. The fraction of the thermal energy that actually goes into heating what you want to heat is given by the efficiency:

$$\text{Energy used} \times \text{Efficiency} = \text{Energy gained}\,. \tag{5.15}$$

The larger the efficienty, the more heat you get and the less is wasted. Equation (5.15) can be re-arranged as follows:

$$\text{Energy used} = \frac{\text{Energy gained}}{\text{Efficiency}}\,. \tag{5.16}$$

Be careful using these two formulas. It is easy to multiply by the efficiency when you should be dividing, and vice versa.

Example 5.4. *Suppose you want* 100 *kWh of heat and you have a furnace that is* 90% *efficient. How much fuel would you need to burn to get this amount of thermal energy*
You would need to burn enough fuel to release

$$\frac{100\,\text{kWh}}{0.9} \approx 111\,\text{kWh}\,. \tag{5.17}$$

What happens is that you burn enough fuel to release 111 kWh. Ten percent of this amount of energy (11 kWh) are wasted and go out your chimney. The other 90% (100 kWh) is used to heat up your house.

5.5 Exercises

Exercise *5.1:* A 1kW electric heater corresponds to how many BTU/hr?

Exercise *5.2:* Suppose you take a ten-minute shower every day using a shower with a flow rate of 2.5 gallons per minute. Estimate how much energy, in kWh, you use for one shower. Is this a little or a lot?

Exercise *5.3:* The dining hall at College of the Atlantic is roughly 90 ft by 45 ft. The ceiling is 12 feet tall.

1. What is the volume of the air in the dining hall?
2. Suppose it is zero degrees outside and we want to heat the air inside the dining hall to a comfortable temperature of 20. How much energy would be required to heat up all the air in the dining hall from 0 to 20? Express your answer in kWh.
3. The air in the dining hall changes over every hour. How much energy would it take to heat the air in the dining hall for one day? Is this a little or a lot?

Exercise *5.4:* Cottage House, a smallish house at College of the Atlantic, uses around 80 MMBTU per year of heating oil.

1. How many gallons of fuel is this?
2. Convert this to kWh.
3. How many tons of CO_2does burning this oil release into the atmosphere?
4. How many people live in Cottage House?
5. How many tons of $CO_2$is this per person? How many kWh of energy is used per person per day? Are these numbers large or small? Put them into perspective.

Figure 5.2: Cottage House at College of the Atlantic. Image source: College of the Atlantic.

Exercise *5.5:* The Katherine W. Davis Center on COA's campus is heated with fuel oil. In one year, 368 MMBTU of fuel were used.

1. How many gallons of fuel is this?
2. Heating oil in Maine currently costs \$2.70/gallon. How much does it cost to heat Davis for one year? Is this a lot or a little? Put this number in perspective.
3. How many tons of CO_2 does burning this oil release into the atmosphere? Is this a lot or a little?

Exercise *5.6:* Suppose you want 100 kWh of heat to keep your house warm on a cold Maine day.

1. If you generate this heat with a traditional electric heater, how much CO_2 is released as a result? How much would it cost?
2. If you generate this heat with a furnace burning heating oil and the efficiency of the furnace if 83%, how much CO_2 would be released? How much would it cost?

Figure 5.3: The Seafox dormitory at College of the Atlantic. Photo credit: College of the Atlantic.

Exercise *5.7:* In 2010–the Seafox dormitory at the College of the Atlantic used 4120 gallons of heating oil. Around 25 students live in Seafox.

1. How much thermal energy is this in BTUs? In kWh?
2. Express this rate of energy use in kWh/p/d. Put this number in context. Is it a little or a lot?
3. How much carbon dioxide was be released into the atmosphere by burning this fuel? Put this number into context. Is this a little or a lot?

Exercise *5.8:* 1. How much energy does it take to heat the water for a typical bath? Answer in joules, BTUs, and kWh.
2. If you used an electric hot water heater that was 90% efficient[10], how much would this cost in Maine? How much CO_2 would be released into the atmosphere as a result.
3. If you used a propane hot water heater that was 70% efficient, how much propane would you need to use? How much would this cost? How much CO_2 would be released into the atmosphere as a result?

[10] This is apparently the minimum efficiency for new hot water heaters. See http://smarterhouse.org/water-heating/replacing-your-water-heater.

Part II

Financial Mathematics

6
The Time Value of Money

Add a paragraph introducing this section. Why are we talking about finance? Well, the hard part of most renewable energy projects is paying for them. There are lots of great things one can do with currently existing, standard technology. Many of these technologies will save a lot of money, but not all at once. They may have a large up-front cost and then produce savings over a period of many years. Should you put solar panels on your roof? Improve the insulation in your house? Get a better water heater or an electric car? Ideally you would do all of these these. But alas, time and money are limited, so we have to choose? How can we decide among several appealing options?

The goal of this part of the book is to teach you some financial mathematics that can help you make these decisions. You'll learn a framework that will enable you to determine the financial value of different investments. This framework and vocabulary is one that is commonly used by investors and CFOs. You'll need to speak this language if you want to go to banks or boards of directors to seek funding for your projects.

6.1 A Thought Experiment

Example from class: a machine that produces $1000 every year, guaranteed, forever. How much would you pay for this machine? Write a few paragraphs about ways to think about this and what issues come up when doing so.

6.2 Time Value of Money

One of the key issues that arises when thinking about the example in the previous section is that money in the future is not as valuable as it is today. This general phenomenon is known as the *time value of money*. If given the choice between being given $100 today or $100 two years from now, everybody would select the first option. What if the two options were \$100 today or \$120 two years from now. There obviously isn't a right or wrong answer to this question—it depends on your circumstances. How badly do you need that money now? And what would you do once you got it? In this section we introduce some important tools that can help us make decisions like this.

Suppose you got the \$100 today and invested it in a bank account that earned 5% interest every year. After one year, you would have

$$\text{Value after one year} = 100(1.05) = 105\,. \tag{6.1}$$

After two years you would have,

$$\text{Value after two years} = 105(1.05) \approx 110\,, \tag{6.2}$$

where we've rounded to the nearest dollar. The 105 in this equation arose because we multiplied 100 by 1.05 in Eq. (6.1). So we can write

$$\text{Value after two years} = 100(1.05)^2 \approx 110\,. \tag{6.3}$$

After two years, the initial 100 has been multiplied by 1.05 twice. And after t years, the initial 100 gets multiplied by 1.05 t times:

$$\text{Value after } t \text{ years} = 105(1.05)^t\,. \tag{6.4}$$

We'll return to this equation in a moment. Let's first go back to the the conundrum: would you rather have \$100 today or \$120 in two years. According to this analysis, you would be better off waiting two years and taking the \$120. Why? Because we saw in Eq. (6.3) if you accepted the \$100 now and deposited it in the bank account it would grow to \$110 in two years—less than the \$120 you could get by simply waiting.

Of course this analysis assumes that if you got the \$100 you would put it in the bank, or do something with it that would make its value grow by five percent a year. If you were starving, you would use the \$100 to buy food right away, and not invest it. The point is that this sort of financial analysis makes sense in the context of investment, not survival.

6.3 *Future Value of a Payment Today*

We return now to Eq. (6.4). It tells us how much \$100 will be worth in t years if we invest those hundred dollars in a bank account or some investment that grows at 5% a year. We can generalize this result as follows:

$$\text{FV} = \text{PV}(1+r)^t\,. \tag{6.5}$$

In this equation PV is the present value of a payment and FV is the future value—how much it would be worth t years into the future assuming an interest rate of r.[1] Let's do a few examples to see how this equation can be used.

[1] Does using FV and PV as variable names bother you as much as it bothers us? We're sorry. It seems to us to be perverse to use *two* letters for *one* variable. Unfortunately, this notation is quite standard. And this sort of thing is a common occurrence in economics, where combinations of letters are frequently used for single variable.

Example 6.1. *Suppose you invest $1000 in an investment that earns* 4% *a year. How much money would you have in* 10 *years. What if the interest rate was* 5% *instead of* 4%*?*
Using Eq. (6.5), we have

$$FV = 1000(1.04)^{10} \approx 1480 . \tag{6.6}$$

If the interest rate was 5%, then:

$$FV = 1000(1.05)^{10} \approx 1629 . \tag{6.7}$$

Note that a fairly small difference in the interest rate r makes a large difference in the future value.

Example 6.2. *In* 25 *years you wish to have $100,000 to use to help your daughter go to college. How much should you deposit in a bank today in order to do this? Assume an interest rate of* 5%*.*
In this problem we are given the future value; we want to have a bank account that is worth $100,000 twenty-five years in the future. What we are looking for is the present value. We can solve Eq. (6.5) for PV. To so, we divide both sides of the equation by $(1+r)^t$, obtaining

$$PV = \frac{FV}{(1+r)^t} . \tag{6.8}$$

Then, plugging in, we find

$$PV = \frac{100,000}{(1+0.05)^{25}} \approx 29,530 . \tag{6.9}$$

So $29.530 deposited in a bank account today would grow to $100,000 in twenty-five years, if you were fortunate enough to have a bank account that paid 5% interest annually.

6.4 *Present Value of a Future Payment and the Discount Rate*

Equation (6.5) tells us the future value of a pile of money (or whatever) that has a value of PV today. As we saw in Example 6.2, above, we can take this equation and rearrange it to isolate PV. Doing so, we obtain:

$$PV = \frac{FV}{(1+r)^t} . \tag{6.10}$$

We use this equation to tell us the present value of a payment that we receive t years in the future and which has a value of FV.

Equations (6.5) and (6.10) look quite similar, and mathematically are equivalent, in that one equation can be algebraically transformed into the other. But they represent different points of view. The equation for future value is thinking forward, from the current moment into the future t years away. In this context, the rate r is usually thought of as the interest rate on an account or the rate of return on an investment.

Equation (6.10) moves in the opposite direction. It takes a future payment of FV and transforms it backwards through time to the present.

In this context, the rate r is usually referred to as the *discount rate*. That is, r is the amount by which future payments are discounted.

There are two important things to bear in mind about the discount rate. First, the present value is an exponential function of time, where $1 + r$ is the base of the exponent. This means that small changes in the interest or discount rate can make a very big difference. The same is true about the future value. The future value of your bank account depends quite strongly on the interest rate r.

The second important thing about the discount rate is that it is a made-up quantity. Where does the discount rate come from? Why might we sometimes use five percent and other times use ten percent? The answer is that the discount rate is, in large part, a heuristic—a quantity that we use to help us think through how we made tradeoffs between the present and the future. [2]

Cite (Richter, 2014, Chapter 7) comments about discount rate.

[2] Use the term *productivity of capital* and discuss. **TODO!**

Example 6.3. *You are to receive a payment of* 4000 *dollars five years from now. What is the present value of that payment if the discount rate is five percent? What is the present value of the discount rate is ten percent?*
We will use Eq. (6.10):

$$PV = \frac{FV}{(1+r)^t} = \frac{4000}{(1+r)^5}\,, \tag{6.11}$$

where we've plugged in $FV = 4000$ and $t = 5$. For $r = 0.05$, I get $PV \approx 3134$. If the discount rate is ten percent, then $r = 0.10$ and we obtain $PV \approx 2484$.

6.5 *Doubling Time*

How long does it take for an investment to double? That is, when does $FV = 2PV$? Let's plug this in to the formula for future value:

$$2PV = PV(1+r)^t\,. \tag{6.12}$$

We need to solve this equation for t. First, we divide both sides of the equation by PV to obtain

$$2 = (1+r)^t\,. \tag{6.13}$$

Next, take the natural logarithm of both sides.[3] Doing so, we obtain

$$\ln(2) = \ln((1+r)^t)\,. \tag{6.14}$$

Using the properties of logarithms, we solve for t:

$$t = \frac{\ln(2)}{\ln(1+r)}\,. \tag{6.15}$$

This is an equation for the doubling time of an investment. The variable t is how long it takes an investment to double in value, given an interest rate of r compounded annually.[4] This equation is exact.

[3] If you're not familiar with logarithms, it's not the end of the world. We'll end up with an approximate formula that doesn't involve a log.

[4] We should briefly mention different types of compounding somewhere.

There are some approximate forms for Eq. (6.15) that are very useful. We will mention one of them here. The doubling time t can be well approximated by

$$t \approx \frac{72}{R} \,. \tag{6.16}$$

This formula is known as the *rule of* 72. In this formula, we've used a capital R to indicate that the rate is plugged in as a percent. So if the interest rate was 5 percent, $R = 5$, not $r = 0.05$. The use of this formula and the exact result, Eq. (6.15) is illustrated in the following example.

Example 6.4. *How long would it take for your investment to double with an annual interest rate of 6%? Calculate the doubling time using the exact formula, Eq. (6.15) and the rule of 72.*
First, let's use the exact formula for the doubling time t:

$$t = \frac{\ln(2)}{\ln(1+0.06)} \approx 11.9 \,. \tag{6.17}$$

So using the exact value we see that the investment doubles in around 11.9 years. Let's see what we get using the rule of 72, Eq. (6.16):

$$t \approx \frac{72}{6} = 12 \,. \tag{6.18}$$

So we see that the two equations give very similar results.

Add discussion of rule of 72 versus rule of 70. **TODO!**

6.6 *Non-financial Discounting*

Just a placeholder for now. Discuss the social cost of carbon.

- `http://grist.org/article/discount-rates-a-boring-thing-you-should-know-about-with-otters/`. Looks like an excellent article.
- `https://www3.epa.gov/climatechange/EPAactivities/economics/scc.html`.
- `http://green.blogs.nytimes.com/2012/09/18/the-social-cost-of-carbon-how-to-do-the-math/?_r=0`
- Johnson and Hope, The social cost of carbon in U.S. regulatory impact analyses: an introduction and critique. `http://link.springer.com/article/10.1007%2Fs13412-012-0087-7`.
- Stern vs. Nordhaus on cost of carbon and different discount rates.
- Would be interested to see what Frank Ackerman has written or has to say about this.

6.7 *Exercises*

Exercise *6.1:* In 15 years you wish to have $200,000 to put toward buying a house. How much money should you deposit in a bank today to do so? Assume an interest rate of 5 percent. How much money would you need to deposit if the interest rate is 7 percent?

Exercise *6.2:* Suppose that in fifty years someone will give you a million dollars. What is the present value of this gift?

Exercise *6.3:* You deposit 1,000 Euro in a bank account with an interest rate of 1 percent. How much money do you have in six years? How long does it take to double your money?

Exercise *6.4:* Would you rather receive $1000 today or $1300 four years from now? Explain briefly.

Exercise *6.5:* You are considering spending some money today that will save you 1,000,000 pesos in four years. What is the value of this savings today? Assume a discount rate of 5 percent.

Exercise *6.6:* You invest $15,000. Assume your money grows at 4% annually.

1. How much money do you have in ten years?
2. How long would it take your money to double?

Exercise *6.7:* Someone will give you $10,000 in ten years.

1. What is the present value of this payment? Answer this question using discount rates of 5, 10, and 15%.
2. Repeat the above analysis, but assume that the payment comes in 100 years and not 10 years.

Exercise *6.8:* Fill in the missing steps between Eq. (6.14) and (6.15).

Exercise *6.9:* Calculate the doubling time using both Eq. (6.15) and the rule of 72 for each of the following interest rates:

1. 2%
2. 4%
3. 6%
4. 8%

Exercise *6.10:* 1. In one year you will receive a payment of \$2000. What is the present value of this payment?

2. In two years you will receive another payment of \$2000. What is the present value of this payment?
3. In three years you will receive yet another payment of \$2000. What is the present value of this payment?
4. What is the total present value of all three of these payments?

Exercise *6.11:* Let's return to the example that began this chapter—the value of a machine that gives you \$1000 every year forever. Use a spreadsheet or write a program to determine the value of:

1. The first 10 payments
2. The first 100 payments
3. The first 1000 payments
4. The first 10000 payments

What do you think happens to the present value of the machine as we account for more and more payments?

Exercise *6.12:* Add problem where students derive the rule of 70.

7
Valuing and Comparing Investments

In this chapter we will learn about different ways of valuing or comparing investments.

It is very easy to come up with good ideas for generating renewable energy or decreasing energy consumption. The hard part is choosing among several good options and then securing funding so you can carry out these projects. Should you install solar cells on your roof, insulate and seal your house, buy an electric car? If you're like us, you can't afford to do all of these things at once, even if you'd like to. How can you choose?

Surely one set of considerations will be financial. If you insulate your house you will lower your energy bill. But by how much? You will enjoy these savings forever—or at least as long as you own your house. But as we have seen in the previous chapter, money in the future is not as valuable as money today. How can we account for this?

There are a number of common metrics that are used to evaluate and compare investments. In this chapter we will introduce these metrics and give you ways to think about what they mean. It is our opinion that these metrics each have strengths and weaknesses, and we will lay these out. The terms and techniques we introduce here are very standard in the world of investing, and are used not just when working with sustainable energy. In order to get a project funded, you will inevitably need to talk to funders—bankers and venture capitalists and the like. To do so effectively, you will need to be able to carry out these analysis and speak their language.

7.1 Two Examples

Here are two examples of businesses that we will use to motivate the analysis that follows. These examples are obviously too simple and not quite realistic, but they will be useful for illustrating the financial metrics that we'll develop below.

Demeo's Donuts

Anna is thinking of opening a donut business. The business will cost \$20,000 to start up. She'll need to buy donut-making machines and such. Once the business is up and running, she expects to make \$5000 a year for five years. She'll thus make a total of \$30,000.

Figure 7.1: mmmm...Donuts. (Figure source: Rob Boudon. https://www.flickr.com/photos/robboudon/824752133, licensed under Creative Commons-Attribution-NonCommercial-ShareAlike 2.0 https://creativecommons.org/licenses/by-nc-sa/2.0/.)

Brooklyn Bagels

Dave grew up in New York City[1] and has a fondness for bagels. So he is keen to open a bagel business. He expects to make \$2000 in revenue in year one and that his revenue will increase by \$3000 each year after that. So Dave anticipates a revenue of \$5000 in year two, \$8000 in year three, \$11,000 in year four, and \$12,000 in year five. He expects sales to start off slow, because Dave lives in Maine, and Mainers don't necessarily appreciate fine bagels. So it will take a while for word to spread and for his business to take off. Starting the bagel business costs as much as starting the donut business: \$20,000.

[1] He's from Manhattan, not Brooklyn, but Brooklyn is trendy now, so he thought it would be a good name for his bagel business. Also, it alliterates nicely.

The projected revenue streams for the two businesses are shown in Table 7.1.

Table 7.1: Projected revenues for the donut and bagel businesses. Both businesses can be started for an initial investment of \$20,000.

Year	Donut Revenues	Bagel Revenues
1	\$6,000	\$2,000
2	\$6,000	\$5,000
3	\$6,000	\$8,000
4	\$6,000	\$11,000
5	\$6,000	\$14,000
Total:	\$30,000	\$40,000

7.2 *Payback Time*

Figure 7.2: mmm...Bagels. (Figure source: Ezra Wolfe, https://www.flickr.com/photos/ezraw/86861363 licensed under Creative Commons-Attribution-ShareAlike 2.0 https://creativecommons.org/licenses/by-sa/2.0/.)

When considering buying a donut or bagel business—or investing in renewable energy or energy conservation—one important consideration is the *payback time*. This is basically just what sounds like. The payback time is the time needed to earn back the original investment. For the donut business, the payback time is a bit more than three years. After three years, the total revenue Anna will have earned from her business is 18. After four years, she will have earned 24. So between year three and four her revenue will cross \$20,000 and she will have made as much money as she spend. (Recall that the cost of the donut business was \$20,000.

What about the bagel business? Let's see. After three years Dave will have made \$15,000 in revenue and after four years he'll have made

$26,000 in revenue. So as was the case for the donut business, the payback time is somewhere between three and four years. years you'll have $18,000 in revenue, and revenue will reach $30,000 after five years. So the payback time is a bit more than four years. As one would suspect from looking at Table 7.1, the donut business pays for itself a bit faster than the bagel business.

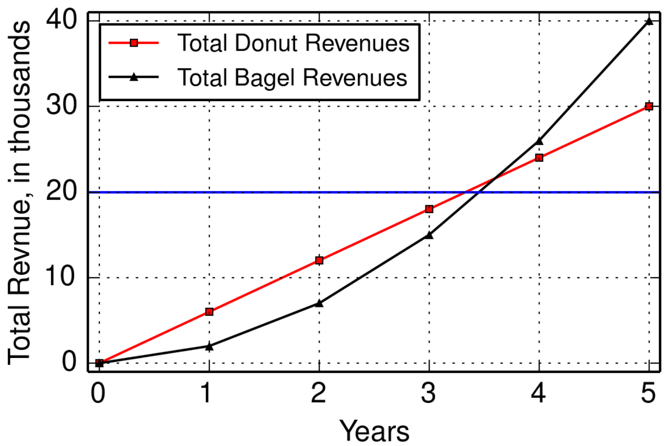

Figure 7.3: The cumulative revenue for the donut and bagel businesses. The revenues were given in Table 7.1. We see that both businesses have a payback time between three and four years.

This situation is shown graphically in Fig. 7.3. The cumulative revenue for the donut business is shown in red and while the bagel business's cumulative revenue is shown in black. The horizontal blue line is the break-even point: $20,000. We can see that the red and black lines both cross the the blue line between years three and four. The red line crosses a bit sooner, but for all practical purposes they cross at the same time. Remember that the revenues in Table 7.1 are just estimates; they're statements about the future and, of course, the future hasn't happened yet, so we can't be sure about it. So the bottom line is that these two investments, the bagel and donut businesses, have essentially the same payback time.

In any event, the payback time is one very common way to evaluate an investment. Benefits of the payback time are that it is simple to calculate, intuitive, and easy to understand. However, it does not take into account the time value of money. Equally important, it doesn't tell you anything about what happens after you have paid back your original investment. After you've broken even, do you make any more money? If so, how much? Or do you expect the investment to stop making money for you shortly after you've broken even, perhaps because the equipment has reached its lifetime? Put another way, the payback time tells you how long it will take you to break even (without discounting) but says nothing about how much money you will make, if any, after the break-even point.

7.3 ROI: Return on Investment

A metric complementary to payback time is the *return on investment*, or *ROI*. Like the payback time, the ROI is basically just what it sounds like: how much money an investment makes divided by the cost of that investment.

$$\text{ROI} = \frac{\text{Gain from Investment} - \text{Cost of Investment}}{\text{Cost of Investment}} \tag{7.1}$$

For the donut business, the cost of the investment is \$20,000 and the return—the total revenue—is \$30,000. Thus the ROI for the donut business is:

$$\text{ROI} = \frac{\$30,000 - \$20,000}{\$20,000} = 0.5\,. \tag{7.2}$$

The ROI is a percent. In this case we would say that the return on investment is 50%. The profit for the donut shop is \$10,000, which is 50% of the cost of the investment. That is, fifty percent of \$20,000 is \$10,000.

The ROI for the bagel business is higher. The bagel shop pulls in more revenue than the donut business, yet both businesses cost the same. The ROI for the bagel business is:

$$\text{ROI} = \frac{\$40,000 - \$20,000}{\$20,000} = 1.0\,. \tag{7.3}$$

So the ROI is 1, or 100%, indicating that Dave's bagel shop will double his money in five years. So if we look only at ROIs, it would seem that the bagel business is a much better investment than the donut shop.

But we hope this seems a bit too simple. The bagel shop makes more money in the long run, but in the first two years, the donut store does better than the bagel shop during the first two year. And due to the time value of money, revenue a few years out is not worth as much as revenue today or next year. So those big bagel-store revenues in years four and five ought to be discounted. The ROI does not account for the time value of money. All that matters in a calculation of ROI is how much revenue you get total, not when you get it. The next two investment metrics we'll encounter account for the time value of money. Before we turn our attention to them, however, a few additional remarks about ROI.

First, the notion of ROI can be extended to account for things other than money. Of note is the energy ROI, in which the accounting is done purely in energy units. For example, suppose we were interested in a wind turbine. To calculate the energy ROI one would measure the cost by determining the amount of energy used to make, transport, install, and maintain the turbine. The gain would be the total energy that the wind turbine would generate over its lifetime

Second, people say ROI and mean all sorts of different things. At issue is what gets counted as a cost and what gets counted as a gain. Also, often ROI is expressed per year, but other times it isn't.

7.4 NPV: Net Present Value

The *net present value*, or NPV, of an investment is the total present value of all future revenue. For example, for the donut investment the NPV would be calculated as follows. First we need to figure out the present value of the revenue of $6,000 that is received one year in the future. To do we need to choose a discount rate. We'll use five percent. Using Eq. (6.10), this is:

$$PV = \frac{\$6,000}{(1+0.05)^1} = \$5,710 \,. \tag{7.4}$$

Next, we need to determine the present value of the $6,000 that is received in year two. Doing so, and again assuming a five percent discount rate, we obtain:

$$PV = \frac{\$6,000}{(1+0.05)^2} = \$5,440 \,. \tag{7.5}$$

Year	Donut Revenues	Present Value	Bagel Revenues	Present Value
1	$6,000	$5,714	$2,000	$1,900
2	$6,000	$5,440	$5,000	$4,540
3	$6,000	$5,180	$8,000	$6,910
4	$6,000	$4,940	$11,000	$9,050
5	$6,000	$4,700	$14,000	$10,970
Total:	$30,000	$25,980	$40,000	$33,370

Table 7.2: Projected revenues and their present values for the donut and bagel businesses. These calculations were done assuming a five percent discount rate. For this discount rate, the NPV of the donut business is $25,980; for the bagel shop it is $33,370.

We could continue on, using Eq. (6.10) to calculate the present values of the donut revenues for years three, four, and five. The result of doing this is shown in the third column of Table 7.2. The sum of all of the present values of the donut revenues is the *net present value*, or NPV, of this investment. For the donut business, the NPV is $25,980.

The NPV is, as the name suggests, the total present value of all future revenues. It is a way of putting a value on an investment. For the donut example, the NPV of $25,980 is the value of the business, assuming a discount rate of five percent. If someone offered to sell us the business for \$22,000, this would be a pretty good deal; we could buy the business for a good bit less than it is worth.

We can also calculate the NPV for bagel business. The present values of all the future revenues are shown in the rightmost column of Table 7.2. Adding up all of these present values yields the net present value of \$33,370 for the bagel business. Note that in both cases the NPV is smaller than the total revenue. The reason for this is that the NPV discounts revenue that we receive in the future. For example, \$8,000 three years from now only has a present value of \$6,910 today, assuming a five percent discount rate.

The NPV depends on what one chooses for the discount rate. The larger the discount rate, the smaller the NPV will be. We will explore this phenomenon in the next section. Doing so will lead us to another important metric that is used for valuing investments.

7.5 IRR: Internal Rate of Return

In the previous section, we saw that if we assumed a discount rate of 5%, the net present value (NPV) of Anna's donut business is \$25,980. Different discount rates give different NPVs. If the discount rate is smaller, the NPV will get larger. The reason for this is that a smaller discount rate means that we are discounting the future payments less, and so the total present value is more. For example, if the discount rate is 3%, the NPV increases to \$27,480. And we were to increase the discount rate, the NPV would decrease. A larger discount rate means we apply more of a discount to future payments, decreasing the total present value of the investment. If, for example, we used a discount rate of 7%, the NPV decreases to \$24,601.

These NPV values, and many more, are shown in Table 7.3. We've calculated the NPV for many discount rates r for both the donut and the bagel investments. We see that as the discount rate increases the NPV decreases.

The cost of both of these investments is \$20,000. For a discount rate of 5%, the NPVs for the donut and bagel businesses are \$25,980 and \$33,370, respectively. In both cases, the NPV is larger than the system cost. The discount rate "punishes" future payments by devaluing them. As we apply a greater and greater punishment to future payments, eventually the NPV decreases until it is equal to the system cost. This defines a quantity known as the *internal rate of return* (IRR). The IRR is the discount rate at which the NPV of the investment equals the cost.

Looking at Table 7.3, we see that for the donut shop, the IRR is between 15 and 16 percent. The NPV at a discount rate of 15% is \$20,110 and the NPV at a discount rate of 16% is \$19,650. The IRR is the discount rate at which the NPV equals \$20,000. So the IRR

Discount Rate	Donut NPV	Bagel NPV
0	$30,000	$40,000
1	$29,121	$38,538
2	$28,281	$37,148
3	$27,478	$35,826
4	$26,711	$34,568
5	$25,977	$33,370
6	$25,274	$32,228
7	$24,601	$31,140
8	$23,956	$30,103
9	$23,338	$29,112
10	$22,745	$28,167
11	$22,175	$27,264
12	$21,629	$26,401
13	$21,103	$25,575
14	$20,598	$24,786
15	$20,113	$24,030
16	$19,646	$23,306
17	$19,196	$22,613
18	$18,763	$21,948
19	$18,346	$21,311
20	$17,944	$20,700
21	$17,556	$20,113
22	$17,182	$19,550
23	$16,821	$19,009
24	$16,472	$18,489
25	$16,136	$17,989

Table 7.3: A listing of the net present value (NPV) for the donut and bagel business for different values of the discount rate r. As the discount rate increases, the NPV decreases. The internal rate of return (IRR) is the discount rate at which the NPV equals the system cost (in this case $20,000).

must be between 15 and 16. For the bagel business, we see from Table 7.3 that the IRR is between 21 and 22%. A more accurate calculation, experimenting with different discount rates, find that the IRR for the donut business is 15.24% and that the bagel's IRR is 21.2%.

Here are a few ways to think about what the IRR tells us. The IRR is a yearly rate of return. If you have an investment with an IRR of, say, 7 percent, then this is an indication that this investment is equivalent to a bank account that earn interest at 7%.[2] The IRR is thus like a yearly ROI, except that it accounts for the time value of money. The larger the IRR, the better the investment. A large IRR indicates a large annual return on your investment. The IRR is probably the most commonly used metric for evaluating investments.

[2] This is illustrated in Exercise 7.5.

To sum up, here is how one calculates the IRR. Choose a discount rate

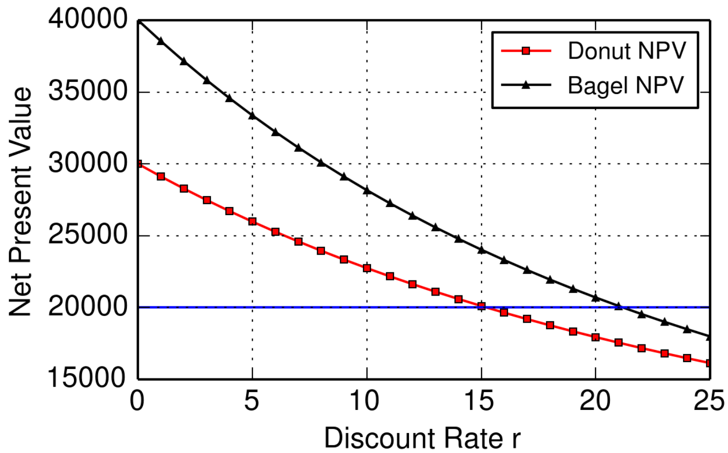

Figure 7.4: A plot of the net present values (NPVs) for the donut and bagel businesses as the discount rate r is varied. The NPV values were given in Table 7.3. As the discount rate is increased, the NPV decreases. The system cost for both the donut and bagel businesses is $20,000. This value is indicated by the horizontal blue line in the figure. The IRR is the discount rate at which the NPV equals the system cost. Graphically, this occurs when the NPV crosses the horizontal blue line. We see that the IRR for the donut business is almost exactly 15% and the IRR for the bagel business is around 21%.

and calculate the NPV for the investment. Experiment with different discount rates until the NPV equals the system cost.[3] The discount rate at which this occurs is the IRR. In all but the simplest cases the IRR cannot be calculated by hand; there is not a simple formula for it. One needs to develop a spreadsheet that calculates the NPV for you, and then

[3] This is a somewhat restricted definition of the IRR. More generally, the IRR is the discount rate at which the present value of all costs and expenses are zero. The idea is that we view the initial expense of the system as a cost—a negative revenue or a negative cost flow. In addition, there may be other negative cost flows, perhaps due to maintenance or labor costs, at any point in the project.

7.6 Cost of Capital and the Hurdle Rate

Determining the discount rate is a subjective thing.[4] Different organizations or people will use different rates depending on their needs. A company (or town or individual) figures out or estimates the rate of return that an investment of similar risk would be expected to make. This rate is often called the **cost of capital**. It is a measure of how expensive it would be to pull capital away from your typical investment. For example, if I am making a rate of return of r by keeping my money in a bank, and then I take this money out of the bank to invest somewhere else, I have lost that interest, and thus would view that lost interest as a cost.

A company will want the rate of return on an investment to be greater than its cost of capital. For this reason the cost of capital is also sometimes referred to as the **hurdle rate**. The rate of return on an investment must be higher (hurdle over) the cost of capital in order for it to be worthwhile.

[4] Where does this section go? Maybe in the previous chapter? I'm not sure.

7.7 Discussion

All of this is nowhere near as objective and scientific as it sounds. There is the matter of risk, which is fundamentally unknowable, especially for new technologies. Risk is often accounted for by using very large discount rates.

How far out should we do our calculations? Five years? Twenty years? Forever? Also, it is not always clear what should count as a cost, even from the point of view of the investor. There are also costs that are born by others. E.g., The burning of fossil fuels creates particulate matter in the atmosphere, leading to asthma. The costs of increased asthma rates—medical costs and lost productivity— are not born by the investor. This is an *externality*—a cost that is not reflected in what the investor has to pay to build and operate the power plant.

7.8 Exercises

Exercise *7.1:* An investment that costs $15,000 has an ROI of 0.8. How much revenue will you get from this investment?

Exercise *7.2:* Verify that the rightmost column of Table 7.2 is correct.

Exercise *7.3:* Suppose that the donut business whose revenue is described in Table 7.1 lasted for seven years instead of five. (Perhaps the donut-making machine is higher quality and will last longer.) Without doing any calculations, determine if the following quantities will increase, decrease, or stay the same. Explain briefly.

1. The payback time
2. The ROI
3. The NPV
4. The IRR

Exercise *7.4:* Suppose that the bagel business described in Table 7.1 has a year-five revenue of $5,000 instead of $14,000. (Perhaps a rival bagel business opens, depriving you of some revenue.) Without doing any calculations, determine if the following quantities will increase, decrease, or stay the same. Explain briefly.

1. The payback time
2. The ROI
3. The NPV

4. The IRR

Exercise *7.5:* This exercise will lead you through a series of calculations designed to help you understand what the internal rate of return (IRR) means. We will analyze an investment that will pay you $3000 in one year and $5000 in two years. You can buy this investment for $7000. It turns out that the IRR for this investment is 8.6%.

1. Verify that 8.6% is indeed the IRR for this investment. To do so, calculate the NPV of the investment and show that it is approximately $7000.
2. If you buy the investment, how much money will you have in two years? To figure this out, you will need to take the value of the $3000 that you get in year one and project it forward, using a discount rate of 8.6%, to determine its value in year two.
3. Suppose that you decide against buying the investment and instead opt to deposit your $7000 in a bank account that makes 8.6% interest. How much money would have in two years?
4. The answers to the above two questions should be the same. Use this observation to write a sentence or two about the meaning of IRR.

Figure 7.5: mmm...scones! (Figure source: OctopusHat, https://commons.wikimedia.org/wiki/File:Pile_of_scones.jpg licensed under Creative Commons Attribution-Share Alike 2.0 Generic https://creativecommons.org/licenses/by-sa/2.0/deed.en.

Exercise *7.6:* You are considering purchasing a scone business. You have good reason to believe that you will make $3000 in profit at the end of the first year, $4000 in profit at the end of the second year, and $5000 in profit the third year. You can buy the business for $10000.

1. What is the payback time of this investment?
2. What is the return on investment?
3. What is the total present value of the three payments if you use a discount rate of 0.05?

Exercise *7.7:* What is the IRR of the scone business described in the previous problem?

Part III

Applications

8
Lighting and Appliances

blah blah

8.1 Incandescent Lights

8.2 Fluorescent Lights and LEDs

8.3 Other Appliances

8.4 Phantom Loads

more blah blah.

Example 8.1. *Statement of example*
Solution goes here

8.5 Exercises

Exercise *8.1:* A questions

1. part a
2. part b

9
Heating I: Leakiness

When it is cold outside we need to heat our homes. The reason is that heat escapes from the inside of our house to the outside. In this chapter we'll think about how and why heat leaves a house. It is this rate of heat loss that determines how much heat we need to provide to a house to keep it at a comfortable temperature. In the next chapter we'll look at different ways of generating that heat—woodstoves, furnaces, heat pumps, and so on.

Heat leaves a home via two mechanisms: warm air escapes through holes in the house, and heat conducts through the walls, windows, and roof. We'll start by looking at escaping warm air.

9.1 Heat Loss Due to Air Leakiness

Let's consider a house that holds a volume of air V, and let ΔT be the different between the outside temperature and our desired inside temperature. For example, it if is -5 degrees Celsius outside and you want the air in your house to be 20 degrees C, then $\Delta T = 25$. How much energy is needed to increase the temperature of a volume V of air by ΔT? To answer this question, we use Eq. (5.1):

$$\text{Heat needed} = mc\Delta T \,, \tag{9.1}$$

where c is the specific heat of air, whose value is approximately 1200 J/m^3 C. In this case, however, we use V instead of m, because the specific heat of air is expressed per volume instead of per kilogram. So I'll rewrite Eq. (9.1) as:

$$\text{Heat needed} = Vc\Delta T \,, \tag{9.2}$$

Warm air is continually leaking out of the house and being replaced by cold air from outside—air that needs to be heated so that its temperature increased by ΔT. The rate at which air leaves the house will be larger for leakier and more poorly sealed homes. We'll express this leakiness in term of the number of air changes that occur every hour. Although this is often abbreviated ACH[1], I'll use N. The leakier the house, the larger N will be. An N of 0.5 means that half of the air in a house changes over every hour. Or, in one day you'll have to heat up the air in your hour 12 times. So smaller air exchange rates are good. But there is a limit to how small we can make N for a house. If N is too small then the occupants of the building won't have enough fresh air to breathe. For typical residences N is between 1 and 2. You might want this to be a bit higher for bathrooms and kitchens, however. I think anything 0.5 or lower would start to be considered unhealthful, both for the house and for the creatures living in it.

[1] Somewhere mention that N can be indirectly measured by a blower door test.

Using the air changes per hour N, I can express the rate at which energy leaves house.

$$\begin{aligned} \text{Rate of heat loss} &= \text{No. of changes per hour} \times \text{Energy to heat air} \\ &= NCV\Delta T \,. \end{aligned} \tag{9.3}$$

Note that the rate of heat loss, being a measurement of the flow rate of energy, is a power.

Let's futz around with units a bit so we can get everything in terms of meters, kilograms, and seconds. This means that our rate of heat loss will come out in units of Watts. We'll need to convert N to exchanges per second. Doing so:

$$N = \frac{N}{1\,\text{h}}\left(\frac{1\,\text{h}}{3600\,\text{s}}\right) = \frac{N}{3600\,\text{s}} \,. \tag{9.4}$$

Using this, Eq. (9.3) becomes:

$$\text{Rate of heat loss} = 1200\frac{\text{J}}{\text{m}^3\,\text{K}}\frac{N}{3600\,\text{s}}V(\,\text{m}^3)\Delta T(\,\text{K}) \,. \tag{9.5}$$

This then simplifies to:

$$\text{Rate of heat loss} = \frac{1}{3}NV\Delta T \,. \tag{9.6}$$

This equation gives the rate of heat loss, in units of Watts, due to warm air leaking out of the house being replaced with cold air from the outside. It is important to remember that this equation is only true if the volume V is measured in cubic meters and the temperature difference is measured in Celsius.

9.2 *Heat Loss Due to Conduction*

Houses also lose heat because it flows out through the walls, roof, and windows. This type of heat transfer is called *conduction*, and is a different mechanism than hot air directly leaving the house through holes in the walls. In conduction, the heat is flowing through the wall itself, just as heat might flow from a hot frying pan into its handle.

The rate of conductive heat loss is given by the following formula:

$$\text{Rate of heat loss} = \frac{A}{R}\Delta T \,. \tag{9.7}$$

In this equation A is the surface area of the house and ΔT is again the temperature difference between the inside and the outside of the house. Let's think about this equation term by term. The larger the area of the wall[2], the more area there is for the heat to flow, and so the greater the rate of heat loss. The ΔT in Eq. (9.7) tells us that the rate of heat loss depends on the temperature difference between the interior and exterior of the wall. In practical terms, the colder it is outside, the faster heat will flow out through the wall.

[2] Eq. (9.7) applies to all surfaces, not just walls. But for the sake of concreteness, I'll refer to the surface as "wall."

The quantity R in Eq. (9.7) is the thermal resistivity. It is a measure of how well insulated the house is. The larger R is, the better the insulation, and the smaller the heat loss. To see this mathematically, note that in Eq. (9.7), R is on the bottom; this means that increasing R will make the fraction smaller, leading to less heat loss. Different materials will have different R values. Metal, which conducts heat very well, will have a small R value. Styrofoam or wool, which conduct heat poorly, have large R values.

What are the units of R? Good question. In the friendly metric SI system, R has units of square meters times degrees Celsius divided by Watts. That is:

$$R = \frac{\mathrm{m}^2\,\mathrm{C}}{\mathrm{W}} \,. \tag{9.8}$$

One way to see this is to note that the rate of heat loss in SI units must be Watts. Then R has to have units of m^2 C/W in order for Eq. (9.7) to be dimensionally consistent. What about American/imperial units? Here things start to get a bit ugly. In imperial units, R is a mess:

$$R = \frac{\mathrm{ft}^2\,\mathrm{F}^\circ\,\mathrm{h}}{\mathrm{BTU}} \,. \tag{9.9}$$

If you go to buy insulation in the US and Canada, its resistance will be given in these units. Since feet-squared-Fahrenheit-hour-per-BTU doesn't exactly roll of the tongue, one almost always refers to an R-value simply with a number. The unwieldy units are understood and need not be spoken. Outside of the US and Canada, the units for R are usually the metric ones: m^2 C/W.

Another way of measuring the insulative value of a material is via its conductivity or thermal transmittance. This is usually denoted U and is just the reciprocal of R. That is, $U = 1/R$. So materials with high thermal resistance have low conductivity. And conversely, materials with low thermal resistance have a high conductivity. In much of the non-North American world, u values are more common the R values. I'm going to proceed in a North American centric way, and use imperial R values. We're not going to do much, if anything, with numerical values for R, so the units aren't going to be crucial.

9.3 Total Heat Loss

Summing up and stepping back, we've got two things going on. Your house is getting colder because hot air escapes and because heat leaves via conduction through the walls and other outward-facing surfaces. These flow rates are captured, respectively, by Eqs. (9.6) and (9.7). Since both of these phenomena are occurring, to get the total rate of heat loss in a house we need to add them together:

$$\text{Total rate of heat loss} = \frac{1}{3}NV\Delta T + \frac{A}{R}\Delta T \,. \tag{9.10}$$

Significant care must be taken when using this formula. The first term, for the heat loss due to escaping air, is true only if N is the number of air exchanges per *hour*, V is measured in *cubic meters*, and ΔT is in *Kelvin*, or, equivalently, *Celsius*. The second term of this equation involves R-values, which in the US are going to be in messed-up imperial units. Rather than transform your R-value to SI units, it is usually easier to turn other quantities into imperial. That is, one has to convert the surface area of the house into square feet and use Fahrenheit for the inside–outside temperature difference.

We would like to minimize the rate at which heat flows out of our house, because then we will need to generate less heat in order to stay warm. This will save us money and reduce greenhouse gas emissions. So how can we make the total rate of heat loss, Eq. (9.10), smaller? There are several possibilities.

- Live in a smaller house. A smaller house will have a smaller surface area A and a smaller volume V, leading to a smaller rate of heat loss. Also, you can make the effective size of your house smaller if you can control heat room-by-room. There is no need to fully heat rooms that you are not in.
- Air seal your house. Make your house less leaky so that hot air escapes less quickly. Mathematically, this corresponds to decreasing N in Eq. (9.10).
- Turn down the thermostat. If you keep your house a bit less warm, then ΔT will be smaller and you will lose heat at a slower rate.

This last point is worth saying a bit more about. Following MacKay, note that both of the terms on the right-hand side of Eq. (9.10) are of the following form:

$$\text{Rate of heat loss} = \text{leakiness} \times \text{temperature difference} . \quad (9.11)$$

Which is larger, heat lost due to escaping air or conducing heat? The answer seems to be that there is not a single answer.[3] There does seem to be agreement that doing some basic air sealing if usually the easiest way to save money and energy. The money needed to do do air-sealing is not large, and the payback time is quick. Improving the insulation of an already-built house can be more expensive and, depending on how your house is built, can be tricky.

[3] http://www.greenbuildingadvisor.com/blogs/dept/qa-spotlight/air-leaks-or-thermal-loss-what-s-worse

9.4 *Degree Days and Heating Loads*

Do I need this section? I'm not sure.

9.5 *A bit more about R Values*

Key point: R values combine like resistors in parallel. So a small region of a wall with a low R can "undo" lots of good insulation (high R) elsewhere. Discuss thermal bridging.

My impression is been that insulating homes—either new homes or retrofitting old homes—requires a good bit of experience and specialized knowledge to do well. It's pretty easy to do a poor job of insulating; doing it really well is hard.

9.6 *Exercises*

Exercise *9.1:* Consider a house that is 2000 square feet. Assume that the ceilings are 8 ft. Let's say that it is 0 degrees C outside and you want the air inside your house to be 20 degrees.

1. Estimate how much energy it would take to raise the air in this house 20 degrees C. Express your answer in Joules, kWh, and BTU.
2. If the air in this house changes every two hours, what is the daily energy needed for heat? How much would this cost in Maine if this heat was electric?

Exercise *9.2:* A 1kW electric heater corresponds to how many BTU/hr?

Exercise *9.3:* Cottage House uses around 80 MMBTU per year of heating oil.

1. How many gallons of fuel is this?
2. Convert this to kWh.
3. How many tons of CO_2 does burning this oil release into the atmosphere?
4. How many tons is this per person? How many kWh of energy is used per person per day?

Exercise *9.4:* For heating in Maine, the average ΔT is 11 degrees C. Suppose the occupants of the Davis Center (see Exercise 5.5) turned down their thermostat by one degree C.

1. How much less oil would Davis use in a year?
2. How much money would Davis save in a year?
3. How much less CO_2 would be released into the atmosphere?

10 Heating II: Sources of Heat

In the previous chapter we looked at the rate and which heat escapes a home. This is important, because the rate at which heat escapes is the rate at which you must supply heat to the house if the temperature is to remain constant. Heat leaves a structure both because hot air escapes and because heat flows through the walls and windows. The contribution from these two effects was capture in Eq. (9.10). Or, more simply, we wrote this in Eq. (9.11) as:

$$\text{Rate of heat loss} = \text{leakiness} \times \text{temperature difference} . \tag{10.1}$$

We considered some strategies for reducing the rate of heat loss smaller: make your house less leaky and turn down the thermostat. In this chapter we'll consider different ways of providing heat to a house.

10.1 A bit about Efficiency

Before we look at particular heating technologies, let's think some about efficiencies. The efficiency e of a process can be thought of as follows:

$$\text{Output} = e \times \text{Input} . \tag{10.2}$$

For example, suppose you want to get 2000 kWh out of a furnace that has an efficiency of 0.8. How much heat would you have to create in the furnace? We can use Eq. (10.2) to obtain:

$$\text{Input heat} = \frac{\text{Output heat}}{e} = \frac{2000\,\text{kWh}}{0.8} = 2500\,\text{kWh} . \tag{10.3}$$

The picture here is that in order to get 2000 kWh of heat in your house, you have to burn enough fuel to create 2500 kWh of heat in the furnace. The difference, 500 kWh, goes up the chimney and does not heat your home. When dealing with efficiencies, as we will be doing often in this chapter, it is easy mistakenly multiply when one should be dividing, or vice-versa. So treat efficiencies with a bit of care.

Another thing to be mindful of is that in this chapter we will be dealing with multi-step processes. For example, we might want to analyze converting natural gas to electricity (in a generator) and then converting this electricity to heat (via any of a number of technologies). We'll have to think about the the efficiencies of *both* processes in the chain. When working with multiple efficiencies, it is helpful to keep in mind that to specify the efficiency of a process, we need to know both the initial and final stages of the process. For example, it makes sense to speak of the efficiency of converting coal into electricity. Simply talking about the efficiency of coal can be ambiguous.

Figure 10.1: An electric space heater for sale at Home Depot. (Figure source: http://www.homedepot.com/p/Lasko-23-in-1500-Watt-Electric-Portable-Ceramic-To 100669059Anelectricheater.)

10.2 Traditional Electric Heaters

Electric heaters take several forms, but all operate on the same basic principle. An electric current is sent through a material with high electrical resistance—something that doesn't conduct electricity very well. The resulting "friction" that occurs when electrons are pushed through this material making frequent collisions with the material's molecules, creates heat which then warms up the house. So an electric heater is basically just a resistor. It's a simple, inexpensive technology. As we'll see, however, they are not great from an economic or CO_2 perspective.

As noted above, there are several different forms of electric heat commonly found in houses. Many houses have electric baseboard heat. These are long, rectangular protuberances running along a wall a few inches above the floor. An example of such a heater is shown in Fig. 10.2. This heater is six feet long and provides 1500 Watts of power. In May of 2016 they were for sale at Home Depot for $54.98. A heater like this would need to be hard-wired into the house's electric system. It requires a 240 volt circuit instead of the North American standard of 120 volts.

Figure 10.2: An electric baseboard heater sold at Home Depot. (Image source http://www.homedepot.com/p/Cadet-72-in-1-500-Watt-240-Volt-Electric-Baseboard 100080894, accessed May 16, 2016.)

Electric space heaters are quite common and come in a variety of shapes and sizes. One spaceheater is shown in Fig. 10.1. This space heater is also 1500 Watts. It plugs into a standard all socket. This heater comes with a remote control, saving on the bother of getting up, walking across the room, and pressing a button if one is too cold or too hot. This heater cost $49.99 at Home Depot in May of 2016. Electric heaters are cheap.

What is the efficiency of an electric heater? One hundred percent! All of the electrical energy the flows through the circuit is converted to heat. When converting from a high-quality form of energy like electricity to a low-quality form like heat, 100 percent efficiency is possible. This seems incredible—what could be better than 100 percent? We'll see, however, that there are technologies that are more than 100 percent efficient at turning electricity into heat.

By the way, an old-fashioned incandescent light bulb is essentially a heater. In these bulbs, current is pushed through a filament with a high resistance. The filament warms up and get so hot that it begins to glow, emitting light. Most of the energy goes into heat; only a small fraction of the energy is turned into light. And this light eventually becomes heat once it hits a surface and is absorbed.

In any event, electric heaters are 100% efficient. But what about the efficiency of taking natural gas and converting it into electricity? MacKay (2009, pp. 150–1) uses 53% for the state of the art natural gas plant and then discounts this by 8% to account for grid losses, arriving at 49%[1]. To keep the calculations simple, I'll round this up to 50%.

[1] $0.53 \times 0.92 \approx 0.49$

So conclusion to draw here is that 100 kWh of chemical energy in natural gas can be turned into 50 kWh of electrical energy in a modern power plant. This 50 kWh can then be turned into 50 kWh of heat in your house, since electric heaters are 100% efficient.

10.3 Fossil-Fuel Furnaces

Modern natural gas furnaces can provide hot air at efficiencies between 80 and 95%. The furnace shown in Fig. 10.3 is known as the Kelvinator KG7SM and has an efficiency of 95%. It can deliver up to 118,000 BTU/hour. In the fall of 2017 you could purchase it from home depot for $1,88.57.

Figure 10.3: A gas furnace for sale at home depot. (Figure source: http://www.homedepot.com/p/Kelvinator-95-AFUE-118-000-BTU-Downflow-Residentia 206511962.)

Older boilers will have an efficiency a good bit less than 95%—perhaps 80% is typical. These are nameplate efficiencies, however, so in practice it seems likely that the efficiency is a bit less, since the furnace might not be operating optimally. Nevertheless, we'll use 80% as a guess at the average efficiency of a natural gas furnace, understanding that some old ones will be much less efficient, while newer models will have efficiencies as high as 95%.

So suppose again that we have 100 kWh of chemical energy in natural gas and we want to heat our house. If we used that chemical energy in a furnace in our house, we could get 80 kWh of heat. Note that this is a lot better than turning the gas into electricity and then using that electricity in an electric heater. Doing this only got us 50 kWh of heat.

10.4 Combined Heat and Power (CHP)

10.5 Heat Pumps

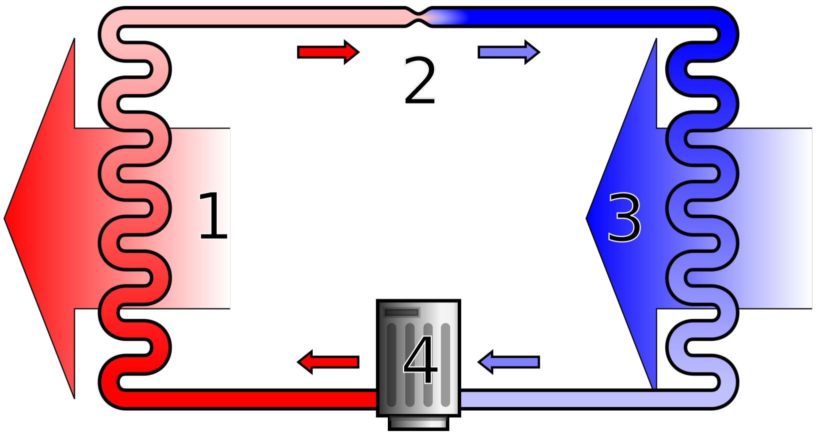

Figure 10.4: A schematic diagram of a heat pump. Image source: https://en.wikipedia.org/wiki/File:Heatpump2.svg. Image released into the public domain by its author, Ilmari Karonen.

Figure 10.5: The interior portion of a heat pump in my living room. The heat pump is the rectangular contraption near the ceiling in between the two windows. Photograph by the author.

The Efficiency for a heat pumps is called the *coefficient of performance* or COP. COPs can be greater than 1. Typical COPs for modern heat pumps are around 4. This means that to get 40 kWh of heat in your house would require only 10 kWh of electricity. So this is a much better

Figure 10.6: The compressor of my heat pump. Photograph by the author.

thing to do with electricity than generating heat by running it through an electric heater.

COPs depend on the temperature, however. The colder it is outside the harder your heat pump has to work to pump heat inside, and thus its efficiency drops.

Heat pumps are "future proof" in that they run on electricity, and so will work for any method of generating electricity.

10.6 Wood

- Is heating with wood carbon-free? Definitely not. Carbon is released due to cutting and transport.

- Is wood low-carbon? Yes. Although how low is in some dispute. The main thing is that burning wood doesn't add any new carbon to the atmospheric cycle the way burning fossil fuels does.

- Can wood be scaled up to heat the world? No.

- Does wood heating made sense for some regions? Almost definitely.

- Efficiencies for woodstoves seem to range from 65% to 85%. Efficiencies for pellet stoves are higher.

- Burning wood in woodstovers may lead to problematic particulate matter. But I suspect this is less of an issue with modern, efficient woodstoves. I think that particulate matter polution is not much of an issue with pellets, since they are burned at a higher temperature and so combustion tends to be more complete. **TODO!** get some references for this.

See https://www.gov.uk/government/publications/an-assessment-of-the-carbon-impacts-of-using-different-ty and https://www.gov.uk/government/publications/life-cycle-impacts-of-biomass-electricity-in-2020

10.7 Air Conditioning

Does this need to be its own chapter? Maybe. Let's see how long this gets with just heating.

10.8 Exercises

Exercise *10.1:* The Kelvinator furnace shown in Fig. 10.3 produces 118000 BTUs per hour. Convert this power to kW.

Exercise *10.2:* Your house needs 75 MMBTU of heat in order to stay warm throughout the entire winter. You have a furnace that burns heating oil. The efficiency of your furnace is 80%.

1. How many gallons of fuel will you need to burn over the winter?
2. How many tons of carbon dioxide will be released into the atmosphere as a result of burning this fuel?

Exercise *10.3:* You have a heat pump with a COP of 3.65. You would like to use your heat pump to get 1100 kWh of thermal energy. How much electrical energy would you need to use to do so?

Exercise *10.4:* The primary source of heat in my house is a Jøtul 500 Oslo woodstove. In a typical winter, I burn 4 cords of wood. The efficiency of the stove is around 74%.[2]. The actual efficiency is surely a bit below this, since the stove isn't operated in the ideal temperature ranges all the time. Let's assume that the actual efficiency is 70%. To answer these questions you will need to use some of the data in Appendix D.

[2] http://jotul.com/us/products/wood-stoves/jotul-f-500-oslo--50528#technical-area

1. How much does this wood cost?
2. How much thermal energy is released by burning 4 cords of wood? We use well-dried hardwood.
3. How much of this thermal energy goes into heating my house?

Figure 10.7: My Jøtul wood stove and three cats. Photograph by the author.

4. How much fuel oil would I need to burn to get the same amount of thermal energy in my house? Assume that the efficiency of the oil furnace would be 80%.
5. How much would this fuel oil cost?
6. How much CO_2 would be emitted by burning this much fuel oil? Put this number in perspective. Is this a little or a lot?

Figure 10.8: Cottage House at College of the Atlantic. Image source: College of the Atlantic.

The next several problems concern Cottage House, a single-family home that is now used as a small dormitory for at College of the

Atlantic. We will use Cottage as a case study for residential heating. Here are some facts and assumptions.

- The average fuel consumption of Cottage House from 2010 to 2014 was 875 gallons. However, the house is unoccupied during December, which is a cold month that would require a lot of heating. So let's say that the average fuel use is 950, so that its consumption is closer to that of a "normal" house and not a dorm.[3]

- I believe that up to six people live in Cottage. But let's say that the occupancy is four, which again makes it closer to a "normal" house and not a dorm.

- The furnace in Cottage is a Milwaukee Thermoflo purchased in 1997. Its nameplate efficiency is 83%. Let's assume that because it is old it isn't operating at full efficiency. Use an efficiency of 80%.

- The average number of degree days per year was 7211 (using a base of 65 degrees). This means that the average difference between the inside and outside temperature was a ΔT of around 20 degrees Fahrenheit, assuming an inside temperature of 65 degrees Fahrenheit.

[3] The State of Maine claims that an average, well-insulated, home of 1500 square feet will use approximately 540 gallons of fuel oil per year. `http://www.maine.gov/energy/fuel_prices/heating-calculator.php`. Cottage is approximately 960 ft^2.

Other useful information can be found in Appendix D.

Exercise *10.5:* **Oil Heat:**

1. How much does it cost per year to heat Cottage per year, assuming a cost of $2.20/gallon for heating oil?
2. How much chemical energy from the heating oil is released when Cottage burns 950 gallons of fuel? Answer in kWh and both kWh per day per person. Is this a little or a lot?
3. How much CO_2 is released into the atmosphere as a result of burning the 950 gallons of fuel oil needed to heat Cottage? Answer in tons of CO_2 per year per person. Is this a little or a lot?
4. Given the furnace's 80% efficiency, how much heat (in kWh) was delivered to the inside of Cottage? Answer in kWh per year and kWh per person per day. This quantity is the *heating load* of Cottage—the amount of heat we need to add to Cottage so it is a comfortable temperature.

Exercise *10.6:* **Resistive Electric Heating.** Suppose that you wanted to generate the heat for Cottage using traditional electric resistive heating.

1. How much would this cost?
2. How much CO_2 would be released into the atmosphere? Assume that we're using average US electricity, that released about 600 grams per kWh of electricity generated. Express your answer in tonnes of CO_2 per person per year.

Exercise *10.7:* **Heat Pumps.** Suppose that you want to generate heat for Cottage by using an electric heat pump with a COP of 4.

1. How much electricity would you need to use in one year to meet the heating load of Cottage? Express your answer in kWh and kWh per person per day.
2. How much would this electricity cost?
3. How much CO_2 would be released into the atmosphere as a result of generating this electricity?

Exercise *10.8:* Comment briefly on the three options you analyzed in the three previous problems: Using an oil furnace, using traditional resistive heating, and using an electric heat pump. Which is the best financially? Which produces the least CO_2?

11 Windpower

Figure 11.1: Wind turbines in Holderness, UK. (Image course: Tom Corses (www.tomcorser.com), Licensed under the Creative Commons Attribution-Share Alike 2.5 Generic license, https://creativecommons.org/licenses/by-sa/2.5/deed.en.

Add a paragraph or two about how wind works by turning a turbine which generates electricity.

11.1 How Much Power is Blowing in the Wind?

How much power is in the wind? A little or a lot? With what efficiency can we extract that power and generate electricity?

Let's picture a wind turbine with diameter D. Air is flowing past the wind turbine with a velocity of v. Let us denote by A the frontal area of the turbine. In a time t how much air moves by the turbine?

$$\text{volume of air in time t} = Avt\ . \tag{11.1}$$

What is the mass of this air? Let ρ denote the density of air.[1] Then the

[1] The density of air is about 1.3 kg per m^3. It varies slightly depending on temperature and humidity.

mass is given by:

$$\text{mass of air in time t} = \rho A v t\,. \tag{11.2}$$

The kinetic energy of this air is given by the usual kinetic energy formula, $E = \frac{1}{2}mv^2$, as discussed in Section 1.3. Thus,

$$\text{kinetic energy of air in time t} = \frac{1}{2}mv^2 = \frac{1}{2}(\rho A v t)v^2\,. \tag{11.3}$$

We are ultimately interested in power, not energy. To get power, we divide by time, since $P = E/t$. This gives

$$\text{power in wind} = \frac{\frac{1}{2}(\rho A v t)v^2}{t}\,. \tag{11.4}$$

Simplifying, this becomes:

$$P = \frac{1}{2}\rho A v^3\,. \tag{11.5}$$

This formula tells us the power in wind of speed v flowing into a wind turbine of area A; the density of air is given by ρ.

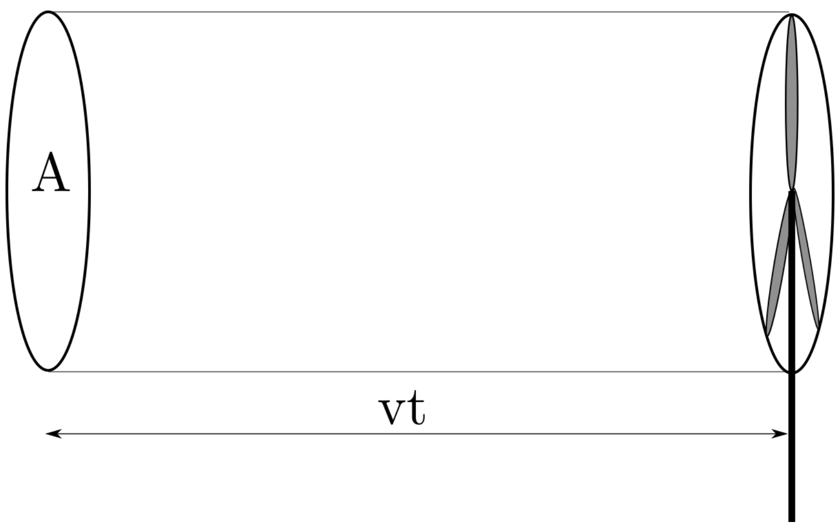

Figure 11.2: A wind turbine with an area of A. In time t the wind in the cylinder flows past the wind turbine. The volume of this cylinder is Avt.

Let's do one more bit of algebra. I'd like to express the area A of the turbine in terms of the diameter d of the blades.[2] The area of a circle is given by πr^2. And the the radius is half the diameter; $d = r/2$. Thus,

$$\text{Area of circle} = \pi r^2 = \pi(\frac{d}{2})^2 = \frac{\pi r^2}{4}\,. \tag{11.6}$$

Putting it all together, we arrive at the following equation:

$$P = \frac{1}{2}\rho\frac{\pi r^2}{4}v^3 = \frac{\pi\rho d^2 v^3}{8}\,. \tag{11.7}$$

This is an expression for the power in wind of speed v that hits a wind turbine that has a diameter of d.

[2] It seems to me like the radius would be more natural to use here, but stating the diameter is a more common way of specifying turbine size. Perhaps this is because the diameter is larger than the radius, and wind turbine companies like big numbers.

11.2 How Much of that Power can we Get?

11.2.1 Betz Limit

How much of the power in the wind can the turbine convert into electrical energy? Surely not all of it, for if that was the case—i.e. if all the kinetic energy of the wind was converted to electricity—then the speed of the wind would drop to zero immediately behind the turbine. But this is impossible. If the wind had zero velocity, the still air would accumulate behind the turbine, blocking the wind. As a result, there would be no wind and the turbine would stop. So it is definitely not possible to convert all of the energy in the wind into electrical energy.

It turns out that the best one can do is to extract 16/27, or around 59% of the kinetic energy in the wind. This result is known as the *Betz limit* or Betz's law. This law follows from basic physics definitions of work, energy, force, and power, as well as conservation of mass.[3,4] So a perfectly engineered turbine could convert at most fifty-nine percent of the power in the wind, given by Eq. (11.7), into electrical power. In practice, turbines might be able to turn 40–50 percent of the wind's power into electrical power. For simplicity's sake, we'll assume, a bit optimistically, that 50 % of the power in wind is converted to electricity.

[3] In a flowing fluid, mass is conserved—for every small region of space, whatever mass flows into that region must also flow out. This type of relationship is often referred to as a *continuity equation*.

[4] A derivation of the Betz result can be found on wikipedia: https://en.wikipedia.org/wiki/Betz's_law. Find a better citation. Perhaps Erlich's new physics of energy book?

With this assumption, Eq. (11.7) becomes:

$$P = \frac{1}{2}\frac{\pi\rho d^2 v^3}{8} \,. \tag{11.8}$$

This expression is the power delivered by a turbine of diameter d in a location with average windspeed v, whereas Eq. (11.7) is the power in the wind that hits the turbine.

Let's step back and think about what this tells us. The power delivered by a turbine depends on three things:

- Power is proportional to the density of the air.
- Power is proportional to the square of the diameter.
- Power is proportional to the cube of the wind speed.

The fact that power is proportional to v^3 is super important, as the following example shows.

Example 11.1. *Suppose a certain wind turbine produces a certain amount of energy per year. What would happen to the yearly energy produced by the turbine if it was moved to some place where it was* 30 *percent windier? What would happen to the energy output if the diameter of the turbine blades was doubled?*

If v increases by 30%, then we replace v by $1.3v$. Since the power from the wind turbine is proportional to the cube of the wind speed, $P = kv^3$, then an increase of 30 % has the following effect:

$$v \to 1.3v\,, \text{ so } P \to k(1.30v)^3 \approx 2.20kv^3 = 2.20P\,. \tag{11.9}$$

In other words, a 30% increase in windspeed leads to an increase in energy production of around 120%; the energy production more than doubles! For a discussion and some examples of working with proportionalities, see Appendix C.

The power produced by a turbine is proportional to the square of the diameter: $P = kd^2$. So if the diameter doubles, we replace d by $2d$. Thus

$$d \to 2d\,, \text{ so } P \to k(2d)^2 \approx 4kd^2 = 4P\,. \tag{11.10}$$

So, a turbine that is twice as large (as measured by blade diameter) will produce four times as much energy.

11.2.2 Capacity Factors

Turbines have a maximum rating. This is the maximum amount of power they can produce under ideal conditions. This is often called called *nameplate capacity*. It is also known as the *load factor*, *rated capacity*, *nominal capacity*, *installed capacity*, or *maximum effect*.[5] Note that "load factor" is the term used by MacKay 2009.

[5] https://en.wikipedia.org/wiki/Nameplate_capacity

The *capacity factor* refers to the actual average power delivered by the turbine. For example, suppose a turbine with a nameplate capacity of 10 MW produced an average power over a year of 2.5 MW. Then the capacity factor would be

$$\text{capacity factor} = \frac{\text{actual average power}}{\text{nameplate capacity}} = \frac{2.5\,\text{MW}}{10\,\text{MW}} = 0.25\,. \tag{11.11}$$

One could also do this calculation in terms of energy. In this case, we would take the actual energy generated in a year and divide that by the maximum possible energy that could be generated in a year—the energy that would be generated if the turbine was operating at full capacity the entire year. But, as always, take care to not mix up energy and power. In the ratio in Eq. (11.11), either both terms should be energy, or both terms should be power. The capacity factor is a ratio; it is a unitless quantity.

A typical capacity factor might be around 1/3, although this can vary. Capacity factors tend to be larger for newer and larger turbines.[6] Capacity factors also tend to be larger for off-shore installations.

[6] http://cleantechnica.com/2015/08/04/wind-could-replace-coal-as-us-primary-generation-s

When the power of a turbine, or any other generating system, is stated without qualification—i.e., a 100 MW solar installation or a 2 GW coal plant—these powers are the nameplate power.

11.3 Where is it Windy?

The wind is stronger higher up. How exactly wind speed depends on height can be a bit complicated.[7] MacKay writes, "as a ballpark figure, doubling the height typically increases wind-speed by 10% and thus increases the power of the wind by 30%. (MacKay, 2009, p. 266)"

[7] Add an experimental graph? **TODO!**

From Ehrlich (2013, Section 7.2.3): Let v_{10} be the windspeed at a height of 10 meters. Then the speed v_h at a height h is given by:

$$v_h = v_{10} \left(\frac{h}{10} \right)^{\alpha} , \tag{11.12}$$

where α is known as the Hellmann exponent. This is an empirical relationship. Different terrains will have different values of α. Ehrlich says that for a flat, unimpeded terrain the value of $\alpha = 0.14$ is often used. In less flat areas where there are house, hills, or trees, αs in the range of 0.3–0.6 are used. **TODO!** come up with an example? I'm not sure how far down this rabbit hole I want to go. I should probably at least mention the Hellmann equation but I don't think I want to do a lot with it.

Additionally, wind is stronger where ever it is unimpeded. This fact is not surprising.

- Ridge-tops are often good.
- Open plains are often good.
- Offshore is often good.
- Taller turbines are better.

11.4 Wind Farms

It turns out that it is best, according to MacKay, to use a spacing of $5d$ between turbines. So far as I know, there is no obvious or simple argument in suggesting why this spacing is best. It is something that was arrived at empirically via trial and error. At issue is the following tradeoff. On the one hand, it is good to have a lot of wind turbines, since each turbine generates electricity. However, the wind speed immediately behind a turbine is slower than the wind speed in front of the turbine. The turbine takes kinetic energy out of the wind. After doing so, the kinetic energy, and hence the speed, of the wind is lower. So if you put turbines too close together, the downwind one will be operating in lower-speed wind and hence will generate less electricity—it would be like putting a solar panel in the shadow of another panel.[8]

[8] Clearly a diagram and a couple of photos are in order here. MacKay has written some about this on his blog. It may be that the spacing of $5d$ that he originally cited is too small.

In any event, as spacing of $5d$ between turbines leads to:

$$\frac{\text{power per turbine}}{\text{area needed for each turbine}} = \frac{\frac{1}{2}\rho \frac{\pi r^2}{4} v^3}{(5d)^2} . \tag{11.13}$$

This simplifies to:

$$\text{wind farm power density} = \frac{\pi}{200}\frac{1}{2}\rho v^3 . \tag{11.14}$$

Note, again, the v^3 dependence. Note also that this doesn't depend on the diameter d. For windspeeds of 6 m/s, the above expression evaluates to a power density of around 2.2 $\mathrm{W/m^2}$.

11.5 Other Issues with Wind

11.5.1 Aesthetics

Some people don't like how wind turbines look. This is a matter of opinion, but they certainly do alter the landscape. But lots of things alter the landscape. For example, agriculture has radically changed the landscape. There are a lot less forests and prairies on the planet than there used to be. This is the price we pay so we can feed the earth's human population. And we think that farms aren't necessarily ugly. Farms can be viewed as enterprises that are essential for providing life for us. So it is with wind turbines. Maybe we can see them as peaceful, life-giving machines.[9]

[9] Add references to Kennedys and others opposing offshore wind turbines because they are visible from shore? Also mention how some people call them farms and others call them industrial wind.

11.5.2 Birds

Do wind turbines kill birds. Yes. Unarguably. The question is not whether or not turbines kill birds, but how many birds are killed. What is the right way to think about this? Let's first talk about a wrong way to contemplate bird deaths.

Figure 11.3: A cat eating a bird on a lawn. (Image source: dr_relling posted on wikipedia https://commons.wikimedia.org/wiki/File:Domestic_cat_eating_bird_on_lawn-8.jpg, licensed under Creative Commons Attribution-Share Alike 2.0 Generic https://creativecommons.org/licenses/by-sa/2.0/deed.en.

It is commonplace to compare turbine-induced bird mortalities with the number of birds estimated to be killed every year by housecats. Typically these estimates indicate that cats are a much greater threat to birds than windpower. This is perhaps amusing, but is not a helpful analysis. For one, if we build more and more turbines, then the number of birds lost to turbines will increase. Are we to conclude from the cat/wind analysis that wind turbines are ok until they kill as many birds as cats, and then wind turbines are bad? This makes no sense.

But more crucially, the issue is that cats do not generate electricity, so it not meaningful to compare cats and wind turbines as sources of bird mortality. A theme of this book is choosing among alternatives. The choice before us is not one of cats versus windpower. Cats and windpower are not in competition with each other. Rather, the issue is, say, do we invest in windpower over solar or nuclear energy. So the sensible way is to compare the harmful effects of windpower (or any other form of power) in terms of mortalities per amount of energy produced.

Two papers by Sovacool (2009; 2013) take a careful look at avian[10] mortality due to wind turbines. By reviewing hundreds of studies, he estimated the number of avian deaths per GWh of electricity generated by nuclear, wind, and fossil fuel. His estimates are shown in Table

[10] This includes bats as well as birds.

11.1. There is some uncertainty in these estimates, and surely more research is needed. But the overall picture seems clear: windpower is not particularly harmful to birds compared to other sources of energy.

Energy Type	Mortality Rate
Windpower	0.269
Fossil Fuels	5.18
Nuclear power	0.416

Table 11.1: Avian mortality rates (fatalities per GWh). Data from Sovacool (2013).

That said, it's worth noting that there is significant variation in avian mortality from wind farm to wind farm, so extrapolating from the analysis of just one or two wind farms can be misleading. A poorly sited wind farm can be a hazard to birds. Some of the first wind farms were not located well, and had a significant impact on birds. Most recent wind farms have been better located and have much less of an impact. Additionally, newer turbines are much safer for birds. They rotate less quickly, and taller turbines tend to pose less of a threat than shorter ones.[11] (The Royal Society for the Protection of Birds (UK) has a sensible statement about wind power that might be worth referencing: `http://www.rspb.org.uk/forprofessionals/policy/windfarms/index.aspx`)

[11] Citation needed. Sovacool?

Finally, it might be worth remembering that the stakes for global climate change are high. If we continue more-or-less with business as usual, most (many?) scientists think we are headed toward a truly catastrophic situation.[12] How many bird deaths are justifiable to lessen the chance of catastrophic climate change? How much of the landscape can we alter in order to prevent N million people from losing their homes due to sea level rise?[13] These aren't scientific questions, of course, but as always we'd argue that these sorts of large, ethical questions need to be informed by facts.

[12] Add citation(s) to how bad the news might be.

[13] Add citation.

11.5.3 Human Health and Annoyances

Are wind turbines annoying? Probably if you live close to one. Are they unhealthy? Highly doubtful.

11.5.4 Intermittency

Critics of wind power like to point out that it is not windy all the time.[14] True. It's not windy all the time. So this is why the grid is important. How much intermittent wind can the grid accommodate? See Chapter 19.

[14] They often point out in the same breath that the sun does not shine all time. It's fine to point this out, but please don't act like you're the first person to think of this.

11.6 Conclusions

Here are the main takeaways from this chapter:

- Power is proportional to v^3. A small increase in average wind speed can make a big difference.
- For terrestrial wind farms: power density ≈ 2 W/m^2.
- For off-shore wind farms: power density ≈ 3 W/m^2.

There is considerable variability in these 2 and 3 W/m^2 figures.[15]

[15] Citation(s) needed.

One important conclusion to draw from the analysis in this chapter is that *bigger is better* for wind turbines. Wind turbines need to be tall, since that is where the wind is windiest. And since the power depends on the cube of the windspeed, this makes a big difference. Put another way, there are economies of scale for wind. A tall wind turbine generates more energy than two turbines half as tall. (See Exercise 11.5).

11.7 Exercises

Exercise *11.1:* Consider Eq. (11.14), which is an expression for the power density of a windfarm. It tells us how much power in watts we could get per meter of a windfarm. What is the power density we would expect from a windfarm in a location where average windspeed is 6 m/s? (The density of air is $\rho = 1.225$ kg/m^3.)

Exercise *11.2:* How much area would it take to generate all of the energy needs of one American from electricity generated on a terrestrial wind farm?

Exercise *11.3:* Suppose that a certain wind turbine generates a certain amount of energy per month. What would happen to the energy generated per month if:

1. The diameter of the blades was increased by 20%?
2. The turbine was re-located someplace where the average wind speed was 30% higher?
3. The turbine was re-located someplace where the density of the substance flowing around it was twice as large?

Exercise *11.4:*

In this problem we'll think some about off-shore wind in the Gulf of Maine.[16]

[16] This exercise makes clear that the spacing in the list environment is highly suboptimal. Dave will work on this someday.

1. First, let's collect some facts: Write down the following values. (All of these figures should be in your class notes. They're also in MacKay and also my book.)

   (a) The average total energy consumption per person in the US, in units of kWh per person per day.
   (b) The worldwide average emissions per person per year, in tons of CO_2 equivalent.
   (c) The average emissions per person per year of the average American, in tons of CO_2 equivalent.
   (d) The average amount of CO_2 released per kWh of electricity generated.
2. What power would be needed to provide the total energy needs of everyone in Maine?
3. Would would be the area needed for an offshore windfarm that could deliver this power. Assume that the offshore windfarm generates 3 Watts per square meter. Express your answer in km^2 and mi^2.
4. What is side of a square whose area is equal to the area you found in the above problem?
5. Find and print out map of New England that includes the Gulf of Maine. Be sure this map includes a scale. Draw on this map a square that is the size of the square you calculated in the previous question. Be reasonably careful when you draw the square, but don't stress out about getting the size super accurate.
6. Assuming that this electricity from wind replaced "average" US electricity, how much CO_2 has been prevented from being released into the atmosphere? Express your answer in terms of tons of CO_2 per Mainer per year. Is this a lot or a little?

Exercise *11.5:*

Suppose that certain wind turbine generates 100 MWh in one year. Approximately how much energy would you expect from two turbines that are identical to the first one, but are half as tall? Explain. Assume that the windspeed increases by 10% when the height doubles.[17]

[17] **TODO!** Re-write this problem so that the diameters of the short turbines are smaller.

Exercise *11.6:* The Vestas V80 wind turbine has a diameter of 80 meters. Let's assume that it operates in a windy location where the typical windspeeds are around 12 m/s. Use Eq. (11.2.1) to estimate the power produced by the turbine. How does the power you calculate compare with the nameplate capacity of 2 MW? More information about this turbine can be found at `http://www.4coffshore.`

com/windfarms/turbine-vestas-v80-2.0-mw--tid53.html.

Exercise *11.7:* A 3MW (nameplate) wind turbine generates 6.6 GWh in one year. What is the turbine's capacity factor? How does this capacity factor compare to typical capacity factors for modern terrestrial turbines?

Exercise *11.8:* The Bingham wind farm will be a 185 megawatt (capacity) wind farm near Bingham, Maine. An article in the *Boston Globe* states that the project is expected to generate enough electricity to power 65,000 homes. [18] Does this seem about right? Explain.

[18] https://www.bostonglobe.com/business/2015/07/01/maine-wind-project-breaks-ground/9ghiUvfC3iDdbzoY7zYImL/story.html

Figure 11.4: Plant Bowen, a coal-fired power plant in Euharless, Georgia. It is one of the largest coal-fired power plants in North America. Figure source: Sam Nash, https://en.wikipedia.org/wiki/File:Plant_Bowen.jpg. Licensed under the Creative Commons Attribution-Share Alike 3.0 Unported license, https://creativecommons.org/licenses/by-sa/3.0/deed.en.

Exercise *11.9:* The Bowen coal-fired power plant has a nameplate capacity of 3.5 GW. In 2006 it generated 22,600 GWh of electricity.[19]

[19] https://en.wikipedia.org/wiki/Plant_Bowen

1. What is the plant's capacity factor?
2. Suppose we wanted to replace the electricity delivered by this coal plant with electricity generated by terrestrial wind power. How large a wind farm would be needed to do so? Come up with a useful way to visualize or conceptualize this area.
3. If this coal plant were shut down, approximately how much CO_{2e} would be prevented from being released into the atmosphere? The lifecycle greenhouse gas emissions for electricity from coal is 1001 $g\,CO_{2e}/kWh$. For electricity from wind, it is

$12 \text{ g CO}_{2e}/\text{kWh}$. [20]

[20] Add citation to IPCC annex.

Exercise *11.10:* This problem concerns the Bull Hill wind project near Eastbrook, Maine. Find the Bull Hill wind project on https://www.eia.gov/state/maps.php.

1. What is the nameplate capacity of the wind project?
2. How much total energy did the wind project generate in 2015? To find this information you'll need to click on "View Data in the Electricity Data Browser."
3. In 2015 what was the capacity of the Bull Hill Wind Project?
4. What was the average power that the project delivered in 2015.
5. Find the Bull Hill wind project on google maps. What area does the Bull Hill project take up? Consider only the 19 western turbines.
6. Computer the power density of the Bull Hill project in W/m^2.

Exercise *11.11:* Repeat problem 11.10 for the Block Island Wind Farm, located south of Rhode Island.

In the next series of problems you will derive the Betz limit, which says that, at maximum, a wind turbine can obtain 16/27 of the power that is in the wind. This derivation follows Ehrlich (2013, Section 7.3).

To begin, consider the air flowing past a wind turbine, as illustrated in Fig. 11.5. We will view air as incompressible. So whatever flows into a region must flow out; air is not created or destroyed, nor can it be compressed or stretched. What this means is that when a column of air slows down it must spread out. This is shown in Fig. 11.5. Air approaches the turbine with a speed v_1. It slows down and flows through the turbine at speed v and continues to slow down and leaves the vicinity of the turbine at a speed v_2.

The same amount of air flows through the turbine (whose area is A) and through the imaginary circles A_1 and A_2. It may help to think of a portion of a river that has no streams flowing into it. In such a river, the flow rate must be the same everywhere. If it wasn't water would accumulate somewhere and it wouldn't be flowing. Thus, where the river is wider the water is flowing slowly, and where the river is narrower the river is flowing more quickly.

We calculated the rate at which air flowed through an area A at speed v back in Eq. (11.2). The rate at which mass flows through A_1, A,

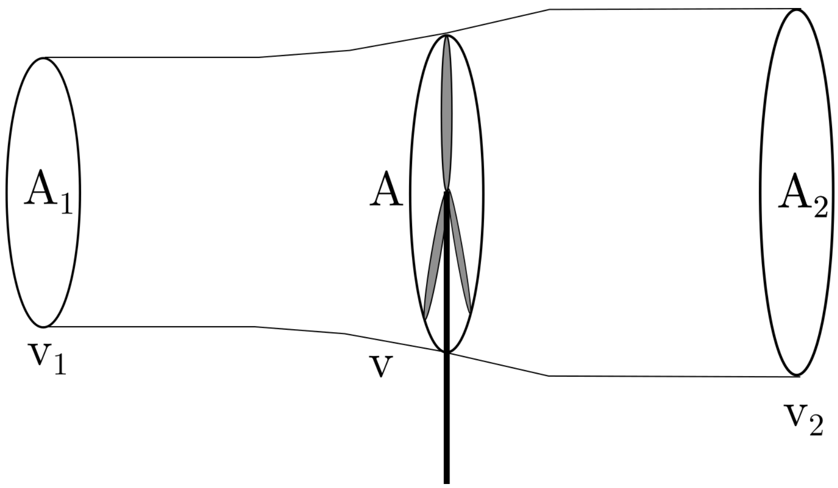

Figure 11.5: Air flowing past a wind turbine. Figure based on Ehrlich (2013, Fig. 7.12). The wind is flowing left to right.

and A_2 must be the same. Thus:

$$\text{mass flow rate} = \frac{dm}{dt} = \rho A_1 v_1 = \rho A_2 v_2 = \rho A v \,, \tag{11.15}$$

where, as before, ρ is the density of air.

Let's now think about forces. Why does the air slow down? It must be that something is exerting a force on the air—Newton's second law[21] tell us that forces are what cause velocities to change. It is the turbine blades that exert force on the wind. We can calculate the Force using Newton's second law

[21] $F_{\text{net}} = ma$.

$$F = \text{mass flow rate}(v_1 - v_2) = \rho A v (v_1 - v_2) \,. \tag{11.16}$$

In the last equation I've used the the fact that the mass flow rate is equal to $\rho A v$, from Eq. (11.15).[22]

[22] **TODO!** Improve derivation of Eq. (11.16).

Now, Power is energy E per unit time. An object's energy changes by the amount of work W done on it, and work is force times distance x, so

$$P = \frac{E}{\Delta t} = \frac{W}{\Delta t} = \frac{Fx}{\Delta t} = Fv \,, \tag{11.17}$$

where I've used the fact that velocity v is $x/\Delta t$. Combining Eqs. (11.17) and (11.16), we have

$$P \rho A V (v_1 - v_2) \,. \tag{11.18}$$

This is the rate at which energy flows out of the wind into the turbine. Equation (11.18) is thus the power that the turbine gets from the wind.

We can come up with another expression for the power that the wind turbine gets from the wind. As argued above, the power lost by the wind is the power gained by the turbine. The power lost by the wind is equal to the difference the rates at which kinetic energy flows through

A_1 and A_2. Using Eq.(11.5), we have:

$$\text{Power lost by wind} \;=\; \frac{1}{2}(\rho A_1 v_1)v_1^2 - \frac{1}{2}(\rho A_2 v_2)v_2^2 \,. \tag{11.19}$$

We now have the ingredients in hand to derive the Betz limit.

Exercise *11.12:* Show that one can use Eq. (11.15) to express Eq. (11.19) as:

$$P \;=\; \frac{1}{2}\rho A v(v_1^2 - v_2^2) \,. \tag{11.20}$$

Exercise *11.13:* We now have two expressions for power, Eq. (11.20) and Eq. (11.18). Equate the right-hand sides of these two equations and derive the following result:

$$v \;=\; \frac{v_1 + v_2}{2} \;;. \tag{11.21}$$

Exercise *11.14:* We next plug Eq. (11.21) into Eq. (11.20). Do so, simplify, and show that one can write

$$P \;=\; C(x)\frac{1}{2}\rho v_1^3 \,, \tag{11.22}$$

where

$$C(x) \;=\; \frac{1}{2}(1 - x^2 + x - x^3) \,, \tag{11.23}$$

and

$$x \;=\; \frac{v_2}{v_1} \,. \tag{11.24}$$

Note that Eq. (11.22) for the power delivered to the wind turbine from the wind is of the following form:

$$P \;=\; C(x) \times \text{power in the wind} \,, \tag{11.25}$$

since we know that the power in the wind is $(1/2)\rho A v_1^3$. Thus, $C(x)$ is the fraction of the power in the wind that goes into the turbine. We'd like to make $C(x)$ as large as possible so as to have the most efficient wind turbine.

Exercise *11.15:* Determine the maximum of $C(x)$. To do so, take the derivative of $C(x)$, set it equal to zero, and solve for x. You'll get two values for x, only one of which is physically meaningful. Plug that value for x into $C(x)$ and you should find that the maximum value for $C(x)$ is $16/27$.

12
Solar Photovoltaic

12.1 How much Energy is in Sunlight?

The sun delivers energy to the earth. At what rate? On a clear day at noon near the equator, the sun delivers around 1000 W per square meter. On average, though, the power hitting the earth is a good bit less. Why? For several reasons:

1. We do not all live on the equator. As one gets closer to the poles, the amount of sun hitting a square meter of earth gets smaller.[1]

2. Most of us live places where it is cloudy at least part of the time.

3. All of us lives places where it is dark for much of the day.[2] This is a phenomenon known as *night time*.

[1] Add simple figure for this?

[2] Except for summers for those who live above or near the Arctic circle.

When we account for all this, we get a quantity known as *insolation*, which is the average power in sunlight actually hitting the earth, per square meter.

Table 12.1: Insolations. Data taken from (MacKay, 2009, p. 46).

Location	Insolation (W/m^2)
London, UK	109
Boston, MA	149
Madrid, ES	177
San Francisco, CA	204
Nairobi, KE	234
Addis Ababa, ET	243
Honolulu, HI	248
Djbouti, DJ	266

So now we know how much power is available to us from sunlight.

12.2 How much of that Energy can we Get?

Solar PV works via the photoelectric effect. An single electron in a semiconductor interacts with a single photon—a particle of light. The electron is loosely bound in the solid. If the photon has enough energy, it can set the electron free. In physics parlance, the electron is promoted to the conduction band. What this means is that it is now free to flow through the material, like an electron is a metal. For conventional silicon solar cells, the amount of energy needed to free one electron is around 1.1 eV.[3]

[3] An "eV" is an *electron volt*. It is a small unit of energy that is commonly used when considering energies for single particles. One electron volt is approximately 1.602×10^{-19} joules.

The spectrum from the sun contains photons of a range of energies, as shown in Fig. 12.1. Any photon that that has an energy less than 1.1 eV cannot set the electron free. Thus, all of the energy from sunlight that is in photons of energy 1.1 eV or less cannot be harvested by the solar cell. Energy in photons whose energy is larger than 1.1 eV can be harvested, but only the part that is left over after the 1.1 eV is used to set the electron free. The net result is that even in a maximally efficient solar cell, not all the energy in sunlight is available to be harvested.

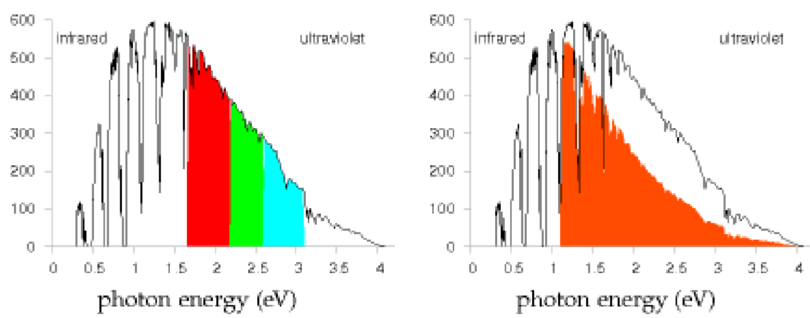

Figure 12.1: The electromagnetic spectrum of sunlight hitting the surface of the earth. The units on the vertical axis are W/m^2 per eV of spectrum interval. For a conventional solar panel with a 1.1 eV band-gap, the orange area on the right figure is the energy that can be captured by the solar panel. Figure source: David MacKay, 2009, p. 47. I don't know if I'm actually allowed to use this figure.

This result is known as the *Shockley–Queisser* limit.[4] For a system with a 1.1 eV cost of freeing electrons, the maximum theoretical efficiency of a solar cell is approximately 32%. In practice, standard solar cells are around ten percent efficient.[5] There are more efficient solar panels, but they are much more expensive, and of these more efficient types of solar are on the market. The vast majority of solar PV presently in are polycrystalline solar panels have an efficiency of around 10%. That is, ten percent of the energy from the sun that lands on the solar cell can be converted into electricity.

[4] Their paper establishing this result was published in 1961 in the Journal of Applied Physics.

[5] Citation(s) needed.

12.3 Capacity Factor and Power Density

The insolations in Table 12.1 range from roughly 100 to 250 W/m^2. If we accounted for the fact that typical solar cells are roughly 10%

efficient, we would expect that the power density of solar PV would be between 10 and 25 W/m^2. In reality, it is a bit lower than this, in large part because in a large solar installation one needs to put some space between the the solar cells so one can access them for maintenance. The highest power densities for solar PV are around 20 W/m^2 in desert PV farms. In Germany, solar installations average around 5 W/m^2. [6] I often use 10 W/m^2 when doing rough calculations.

Average US capacity factor for utility-scale solar PV in 2015 was 28.6%.[7] Typical capacity factors for solar PV in Maine range from 15 to 20%. The higher end range applies to well-sited utility-scale installations.

Let's get a feel for some of these numbers by working through an example.

[6] http://www.theenergycollective.com/robertwilson190/257481/why-power-density-matters, accessed September 21, 2017.

[7] https://www.eia.gov/electricity/monthly/epm_table_grapher.cfm?t=epmt_6_07_b.

Example 12.1. *A ten-panel solar array on the roof of a small barn at College of the Atlantic's Beech Hill Farm has a nameplate capacity of* 2.3 *kW. In 2016 it generated* 3467 *kWh. What is the capacity factor of this array? The area of the barn is approximately* 33 *m*^{2}*. What is the power density of the array in W/m*^{2}*?*

To determine the capacity factor we'll calculate how much energy the array would make in one year if it operated at 2.3 kW of power for the entire year:

$$\text{Energy} = 2.3\,\text{kW} \times 365\,\text{d}\left(\frac{24\,\text{h}}{1\,\text{d}}\right) = 20,160\,\text{kWh}\,. \tag{12.1}$$

The capacity factor is the actual energy generated in a year divided by the maximum possible energy generated in a year:

$$\text{Capacity factor} = \frac{3467\,\text{kWh}}{20,160\,\text{kWh}} \approx 0.172 = 17.2\%\,. \tag{12.2}$$

The power density is expressed in W/m^2. So I need to express the average power of the array in W.

$$\frac{3467\,\text{kWh}}{1\,\text{yr}}\left(\frac{1\,\text{yr}}{365\,\text{d}}\right)\left(\frac{24\,\text{h}}{1\,\text{d}}\right) \approx 0.4\,\text{kW} = 400\,\text{W}\,. \tag{12.3}$$

So the solar cells on the 33 m^2 barn produce power at an average rate of 400 Watts. So the power density is:

$$\text{Power density} = \frac{400\,\text{W}}{33\,\text{m}^2} \approx 12.5\,\text{W}/\,\text{m}^2\,. \tag{12.4}$$

Not bad. This is toward the high end of the power densities for solar PV at our latitude.

12.4 *Rooftop Solar*

A natural place to put solar PV is on the top of roofs. This is unused space[8], so putting panels on our roofs is not a sacrifice or inconvenience. How much electrical energy can we get from our roofs? Would it be enough to power the typical house? Let's do a very rough calculation and see

[8] Unless you spent a lot of time on your roof, I guess.

The mean size of the average home built in the US in 2015 was $2,687$ ft^2 (US Department of Housing & Urban Development, 2015, p. 345). Let's assume that this is a two-story house, so the footprint of the house—and thus the area of the roof—is half of $2,687$. Thus, the area of the roof is 1340 ft^2. Of this, let's estimate that half of the roof area would be available for solar panels. Most roofs are slanted in two directions, only one of which would face south and thus be appropriate for solar. So the effective area of our solar panels will be 670 ft^2. Let's convert this to square meters:

$$670\,\mathrm{ft}^2\left(\frac{1\,\mathrm{m}}{3.3\,\mathrm{ft}}\right)^2 = 62\,\mathrm{m}^2\,, \tag{12.5}$$

where I've used the fact that there are around 3.3 feet in a meter. Note that I need to square the unit conversion factor in parentheses so that the units come out right.

For the power density, I'll use 10 W/m^2. This yields a power of 620 Watts. Let's figure out how much energy these 620 Watts would generate in a month. First, note that 620 W are 0.62 kW. Then,

$$0.62\,\mathrm{kW} \times 30\,\mathrm{d}\left(\frac{24\,\mathrm{h}}{1\,\mathrm{d}}\right) = 430\,\mathrm{kWh}\,. \tag{12.6}$$

What to make of this number? To put things in perspective, recall that the average Maine home uses 520 kWh of electricity a month, while the average home across the US uses 950 kWh a month. So the result in Eq. (12.6) suggests that it will be hard to generate sufficient electricity on your roof in order to meet your home electricity needs. That said, the above estimate was perhaps not very generous. Many locations will be able to achieve a power density of more than 10 W/m^2, and some houses will have more rooftop real estate than the 620 I've allotted.

If I were more generous and assumed that there are 800 ft^2 we can devote to solar panels and that the power density is 15 W/m^2, then I find that the panels would produce almost 800 kWh/month. So this is in the ballpark of what a typical house would use. But in the larger picture, it's worth remembering that residential electricity use is a small fraction of our overall energy use. In the US, we use around 250 kWh per person per day. This includes residential, commercial, industrial, and transportation energy uses of all forms. 800 kWh per month is 26 kWh/day. That's a lot, but still only a bit more than a tenth of our total energy needs.

The picture that emerges is that rooftop solar PV is a significant source of electrical energy. But in order for solar PV to really take a bite out of our total consumption we'll need to do more than just put solar panels on our houses.

12.5 Solar Farms

12.6 Conclusion

1. Solar farms produce around 10 W/m^2. There is considerable variance, however. Locations with higher insolation are clearly better.

2. If you use land for solar, you can't use it for anything else. In contrast, wind turbines can be placed above cropland without affecting the crop yield. An exception, though, is rooftop solar. You can have a perfectly nice house underneath solar panels. But as we've seen, rooftop solar alone isn't going to cut it.

3. Unlike wind, there are no economies of scale. Lots of small installations sprinkled about are ok. For wind, though, this isn't a good idea, due to the fact that the power depends on v^3, and wind is faster as one goes higher up.

12.7 Exercises

Exercise *12.1:* A small town-house style dormitory[9] at College of the Atlantic has a solar array with a nameplate capacity of 8.7 kW. In 2016 it generated 10.4 MWh. The area of the south-facing roof on the house is approximately 125 m^2.

[9] Known locally as Eno House

1. What is the capacity factor of the array?
2. What is the average power produced by the array, in units of Watts?
3. What is the power density of the array, in units of W/m^2?
4. Suppose this solar electricity is displacing electricity generated with a carbon intensity of 500g CO_2/kWh. How much less carbon is emitted into the atmosphere each year as a result of this array? Is this a little or a lot?

Exercise *12.2:* The Limerick nuclear power plant, shown in Fig. 12.2, is a pair of nuclear power stations. Taken together, the two plants generate around 19 GWh of electricity per year.[10]

[10] https://en.wikipedia.org/wiki/Limerick_Generating_Station

1. Express the power of the Limerick stations in watts.
2. What area of solar PV would be needed to generate this amount of power? Assume a power density of 10 W/m^2. Express this area in a sensible way—probably this means figuring out the side of a square whose area is equal to the area of the solar installation.

Figure 12.2: The Limerick Generating Station in Limerick, Pennsylvania, USA. Figure source: Arturo Ramos, https://commons.wikimedia.org/wiki/File:LimerickPowerPlant.JPG.

Exercise *12.3:* In this exercise you'll calculate some energy statistics for Japan. You can get some basic energy facts about Japan on the EIA website: https://www.eia.gov/beta/international/country.cfm?iso=JPN.

1. The total energy consumption is given in terms of a truly horrible unit: Quadrillions of BTUs. These are often referred to as *quads*. One quad is 10^{15} BTUs. BTUs are British Thermal Units, about which the less said, the better. Let's get out of quads and into better units. One quad $= 293 \times 10^9$ kWh. What is the primary energy consumption of Japan per year in units of kWh?
2. What is the *per capita* energy consumption for Japan in units of kWh/person/day? How does this compare to the average *per capita* consumption of the United States?
3. What is the power consumption of all of Japan expressed in watts? This will be a very large and not very meaningful number.
4. What is the power consumption per unit area of Japan? Your answer should be expressed in units of watts per square meter. To obtain this, take your answer to Question 3 and divide by the area of Japan.
5. How does your answer to Question 4 compare to the W/m^2 for solar farms? What fraction of Japan's landmass would have to be devoted to solar farms if it wanted to get all of its energy

from solar? Does this seem feasible?

Exercise *12.4:*

There is a solar installation a few miles northeast of Amalia, New Mexico, USA. In 2014 this plant generated 2997 MWh of electricity.[11] The nameplate capacity of the plant is 1.3 MW.

[11] http://www.eia.gov/electricity/data/browser/#/plant/58240

1. What is the capacity factor of the plant?
2. What is the average power delivered by the plant?
3. Find the plant on google maps. Look at Ventero Road as it heads northwest out of town. Use google maps to estimate the area of the installation.
4. Use your answer to the above two questions to estimate the power density of the solar array, in units of watts per square meter.

Exercise *12.5:* This problem concerns a *Guardian* article[12] about the African Renewable Energy Initiative (AREI).

[12] http://www.theguardian.com/global-development/2015/dec/07/africa-plans-renewable-energy-initiative-solar-hyd

1. The article states that the average annual per person consumption of electricity is around 600 kWh. Convert this to kWh per person per day.
2. The article also states that the worldwide average per capita electricity use is around 3000 kWh per year. Convert this to kWh per person per day.
3. The per capita yearly electricity use in the US is around 13000 kWh.[13] Convert this to kWh per person per day.
4. The *Guardian* article states that AREI has a goal of adding 300 GW[14] of renewable electricity generation by 2030. How much eletricity is this in kWh per person per day? The population of Sub-Saharan Africa is, very roughly, 1 billion.
5. If these 300 GW of power were entirely from solar PV, how much land would this be? Express your answer in an understandable way; perhaps find the size of a square that has this area, or express the area as a fraction of one of the countries in Sub-Saharan Africa.

[13] http://www.iea.org/statistics/statisticssearch/report/?year=2013&country=USA&product=Indicators

[14] It's not quite clear to me if this is nameplate or actual power, but I think it's the latter. So let's assume that this is 300 GW of actual power.

Exercise *12.6:* You are considering buying a solar PV system that would have 20 panels, each with a nameplate capacity of 250 Watts. You expect a capacity factor of 0.15. Purchasing and installing the panels will cost around three dollars per Watt. You expect the system to last for 20 years.

1. How much will this system cost?

2. How much electricity will the system generate over its lifetime?
3. If this electricity was replacing "normal" electricity generated with a carbon cost of 0.5 kg of CO_2 per kWh of electricity, how much carbon dioxide would you have prevented from being emitted?
4. Suppose that there was a price on carbon of $13 per ton.[15] This means that you could get a $13 credit for every ton of carbon that you prevented from being emitted? How much money could you get for carbon credits for your solar PV installation? How does this compare to the cost of the solar panels?

[15] This is the price floor for carbon in the California carbon market. In the EU market the price for carbon has been as low as 5 euro per ton.

13
Hot Water and Solar Thermal

13.1 Hot Water

Estimate for deaily hot water use are all over the place.

13.2 Solar Thermal Hot Water

- Some argue that it is cheaper to heat water with an electric heater powered by solar thermal: `http://www.greenbuildingadvisor.com/blogs/dept/musings/solar-thermal-really-really-dead`. On its face, this doesn't quite seem right, since thermal energy is of such a lower quality than electric energy. But economics and costs don't always make sense.

- I think that a lot depends on whether or not the system is being retrofitted into an existing house, which is expensive and difficult, or if it is built into a new home.

- I've seen installation costs estimated between $8000 and $10,000. I think this is for a retrofit.

- If this was part of a 30-year mortage, it would be a very small addition to your monthly payment.

- We make it easy (via car and home loans) to pay for some things we can't afford. How can we do this for other things, like solar panels and solar thermal?

Example 13.1. *Statement of example*
Solution goes here

13.3 Exercises

Exercise *13.1:* A questions

1. part a
2. part b

14
Cars and Transportation

14.1 Some Basic Car Facts

Cars convert the chemical energy of gasoline into kinetic energy. Later in this chapter we'll explore this process in more detail. For now, let's just focus on some basic facts.

Cars use gasoline. How much? It depends on the car and how one drives it. Are you driving at a constant speed on the highway or are you starting and stopping frequently in a city or town? We'll start by keeping things simple and just use 30 miles per gallon for the typical rate of fuel consumption. [1] There is a lot of variation, but 30 mpg seems like an ok average. In the US the average is more like 25 mpg[2] but 30 is probably a better figure for Europe. Outside of the US fuel efficiencies are typically expressed in terms of how many liters of gasoline are needed to drive 100 km. For more, see Question 14.1, below.

[1] How are fuel efficiencies measured outside the US? Liters per kw?

[2] http://www.autonews.com/article/20150604/OEM05/150609925/average-u.s.-mpg-edges-up-to-25.5-in-may

Figure 14.1: My old car, a 2003 Toyota Corolla. According to www.fueleconomy.gov, its combined city/highway mileage was 31 mpg.

Gasoline, when ignited, produces thermal energy. How much? One liter of gasoline produces 10 kWh of energy. If you prefer gallons: igniting one gallon of gasoline produces 38 kWh. Lastly, gasoline, when burned, produces CO_2, along with some other unpleasant pollutants.

How much CO_2? One kWh of energy from gasoline yields 240 grams of CO_2.

You may initially be surprised that one kWh of energy from gasoline yields only 240 grams of carbon dioxide, while generating a kWh or electricity produces, on average in the US 500 grams of carbon dioxide. Why is this? First, recall that 500 grams is an average over different ways of generating electricity: primarily nuclear, hydro, and various fossil-fuel power plants. For comparing with burning gas in cars, let's focus on fossil-fuel-generated electricity. The average carbon intensity is: 1001 (coal), 840 (oil), and 469 (natural gas)[3] in units of grams of carbon dioxide per kWh of electricity. All these figures are a lot less than the carbon that results from getting one kWh of thermal energy from gasoline.

[3] Figures from Table A.II.4 of Moomaw et al. (2011).

The reason is that when we burn gas we are getting thermal energy, which is of a much lower quality than electrical energy.[4] Thermal energy is not of high quality—it is not all readily available to us. We can use the hot air produced by burning a bit of gasoline to charge our cell phones. Electricity, on the other hand, is very high quality energy. Almost all of the energy in electricity is available to us to do useful things.

[4] Where should I talk about energy quality? Probably somewhere in part I, maybe even in the very first energy chapter.

In any event, let's use these facts to do a simple example so we can start to get a feel for these numbers:

Example 14.1. *Suppose you drive a typical car for two hours at* 60 *mph. How much energy does this take? How much carbon dioxide is emitted by the car during these two hours? How much carbon dioxide would be produced if you did this amount of driving every day for one year?*

First lets figure out how far the car goes.

$$2\,\mathrm{h} \;=\; 2\,\mathrm{h}\left(\frac{60\,\mathrm{mi}}{1\,\mathrm{h}}\right) \;=\; 120\,\mathrm{mi}\,. \tag{14.1}$$

Note that I'm essentially using the speed to convert from hours to miles. How much fuel does this use? Assuming that the car gets 30 miles to the gallon:

$$120\,\mathrm{mi} \;=\; 120\,\mathrm{mi}\left(\frac{1\,\mathrm{gal}}{30\,\mathrm{mi}}\right) \;=\; 4\,\mathrm{gal}\,. \tag{14.2}$$

Next, using the fact that 38 kWh are produced if we burn on gallon of gasoline:

$$4\,\mathrm{gal} \;=\; 4\,\mathrm{gal}\left(\frac{38\,\mathrm{kWh}}{1\,\mathrm{gal}}\right) \;=\; 152\,\mathrm{kWh}\,. \tag{14.3}$$

Let's now turn our attention to carbon dioxide. One kWh of thermal energy from gasoline is responsible for 240 g, or 0.24 kg, of CO_2. So, the 152 kWh we use when driving yields:

$$152\,\mathrm{kWh} \;=\; 152\,\mathrm{kWh}\left(\frac{0.24\,\mathrm{kg}}{1\,\mathrm{kWh}}\right) \;=\; 35\,\mathrm{kg}\,. \tag{14.4}$$

If we did this every day for a year:

$$35\,\frac{\text{kg}}{\text{d}} = 35\,\frac{\text{kg}}{\text{d}}\left(\frac{365\,\text{d}}{1\,\text{yr}}\right) = 13300\,\text{kg}\;. \tag{14.5}$$

So this amount of driving would result in around 13 tons of CO_2 being emitted into the atmosphere in one year.

Whoa. That's a lot of energy and carbon. Equation (14.3) tells us that this amount of driving uses 152 kWh. This is more energy than the average UK resident uses in a day for everything. The 13 tonnes of carbon dioxide is also huge. The average emissions worldwide is around 5.5 tonnes of CO_2 per person per year. So this amount of driving already puts you at twice the world average. And remember that our goal is not to make US residents average but to reduce worldwide emissions to essentially zero.

This has been a fairly rough calculation, and not everyone in the US drives this amount daily. But we hope the point is clear. Driving conventional gasoline cars uses a lot of energy and produces a lot of carbon dioxide. The numbers add up quickly.

14.2 A Basic Model for Car Energy

The above discussion gave some basic facts about cars: how much gasoline is needed to make them go a certain distance (miles per gallon), how much energy is released when burning gasoline (kWh per gallon), and how much Co_2 is released when doing so (kg of CO_2 per kWh). But where does this energy go? In this section we'll develop a simple model of car physics: why does driving a car take the amount of energy it does, and what (if anything) could be done to make cars use less energy.

The chemical energy in the gasoline is converted into thermal energy when it is ignited inside an engine's pistons. This thermal energy is then converted into kinetic energy and the car moves. Physics tells us that objects in motion tend to say in motion. So once the car is rolling along, we should be all set, right?

As you know, it's not so simple. From time to time we need to stop our car to avoid hitting someone or because there is a red light or a stop sign. Also, friction is constantly exerting a force on your moving car, continually sucking away kinetic energy—energy that has to be replaced by your car's engine if you want to keep moving. There are two main types of friction: air resistance and rolling resistance. The latter is the friction associate with your wheels and tires. This tends to be much smaller than air resistance, so we'll ignore it for now.

So let's build up a simple model of the energy use of a car. We'll look at the energy lost do to stopping and starting, and the energy lost due to air resistance. This analysis closely follows that in Chapter A of MacKay (2009).

We'll start by thinking about stopping and starting. Let's picture that you wish to drive your car at an average speed of v and that the average distance between stops is d. This situation is illustrated in Fig. 14.2. What, then, is the average time between stops? Distance equals rate times time, $d = vt$, so

$$\text{Ave. time between stops} = t = d/v\,. \tag{14.6}$$

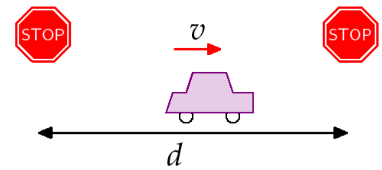

Figure 14.2: A car drives a distance d between stops. (Image source: David J.C. MacKay 2009, Fig. A.2, p. 254.)

Every time you stop, all of the car's kinetic energy goes into the brakes. The kinetic energy of the car is $(1/2)mv^2$, where m is the mass of the car. So, the rate at which energy flows into the brakes is

$$\text{Power going to brakes} = \frac{\text{kinetic energy of car}}{\text{time between stops}} = \frac{\frac{1}{2}m_c v^2}{d/v} = \frac{\frac{1}{2}m_c v^3}{d}\,. \tag{14.7}$$

This equation is the power needed to make the car go, given that you wish to travel at speed v and that the average distance between your stops is d. This is just part of the power that your car needs. The additional power is needed to overcome air resistance, which we discuss next.

As your car moves through air, air resistance exerts a force on the car. This force acts to slow the car down, requiring additional energy from the engine to keep the car moving at a constant speed. We'll think about air resistance in terms of energy and not forces. Suppose you get your car moving nice and fast down a flat highway and then put the car neutral. What happens? You'll gradually coast to a stop. Where has that energy gone? Not into the brakes, since you haven't used the brakes. The answer is that the kinetic energy of your car has been turned into the kinetic energy of the air left swirling in the car's wake.

Here's a simple way to think about this. As the car moves through the air, it needs to push air out of its way to make room for itself. Similarly if you are running through a very crowded room—perhaps you are in a busy train station and are running to catch a train—you need to push people out of your way so you can make it through the

crowd. How much air does the car have to move out of its way during some time interval t? During this time interval the car moves a distance vt. So it displaces a volume of air equal to Avt, where A is the frontal cross-sectional area of the car. What is the mass of this air? We convert from mass to volume via the density ρ; the density ρ of air is around 1.225 kg/m^3. So, the mass of the air displaced by the car is $m_{\text{air}} = \rho Avt$.[5]

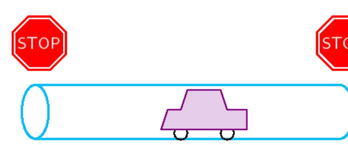

Figure 14.3: A car drives a distance d between stops. In so doing, it needs to displace a volume of air equal to Avt. (Image source: David J.C. MacKay 2009, Fig. A.4, p. 255.) I should perhaps modify this to include vt and A.

[5] Maybe talk about effective area of car versus actual area of car; drag coefficient. Probably should do this here or somewhere.

This is the mass of the air that is moved by the car. How fast is this air going? We'll need this in order to come up with an expression for the kinetic energy of the swirling air. We'll make the assumption that the speed of the swirling air left behind the car is equal to the speed of the car. This is only an approximation, but it seems reasonable. Think about standing next to a road while a car zips by. The faster the car is moving, the faster the wind you'll experience roadside.

With this assumption, we can now write down an expression for the kinetic energy of the swirling air left behind by the car:

$$\frac{1}{2}m_{\text{air}}v^2 = \frac{1}{2}\rho Avtv^2 = \frac{1}{2}\rho Atv^3 . \tag{14.8}$$

This is the kinetic energy lost by the car to swirling air every time interval t. So the power associated with this is

$$\text{Power going to swirling air} = \frac{\frac{1}{2}\rho Atv^3}{t} = \frac{1}{2}\rho Av^3 . \tag{14.9}$$

We can now combine Eqs. (14.7) and (14.9) and come up with an expression for the total power the car is losing to stopping-and-starting and air resistance:

$$\text{Power lost by car} \approx \frac{1}{2}m_c v^3/d + \frac{1}{2}\rho Av^3 . \tag{14.10}$$

This equation gives the rate at which the car is losing energy. The car's engine needs to replenish this energy at the same rate in order to keep the car moving. To do so, the engine converts the chemical energy in the gasoline into thermal energy (heat) and then kinetic energy. This conversion chain is only around 25% efficient. This means that if you burn gasoline and release 100 kWh of thermal energy, only 25 kWh of this can be converted into kinetic energy by the engine. Thus, we can write the power used by the car as:

$$\text{Power used by car} \approx 4\left[\frac{1}{2}m_c v^3/d + \frac{1}{2}\rho Av^3\right] . \tag{14.11}$$

This equation gives the rate at which the car uses gasoline.

This is a very simple picture of a car's energy use, but it gives reasonable results. If you plug reasonable numbers into Eq. (14.11) you'll get numbers consistent with the empirical figures discussed in the previous section. You can explore this in Question 14.8 if you wish.

14.3 Implications of the Car Model

To do better in city driving:

- Make car less massive: decrease v_c.
- Drive more slowly
- Stop less often

- Recover the energy lost to braking.

  To do better for highway driving:

- Drive more slowly
- Decrease the cross-sectional area of the car.
- Decrease the drag on the car.

14.4 Electric Cars

14.5 Planes

What about planes. Planes use gasoline? How much? This varies, but a good average is a plane uses roughly 40 kWh to move one person 100 km. This is also written as 40 kWh/100person-km. This figure takes into account the fact that planes are mostly full. That is, 40 kWh is the single-person share of the total energy needed to move the plane 100 km.

Example 14.2. *Suppose you make two round-trip flights from New York City to San Francisco each year. Approximately how much energy does this take, in kWh/day?*
It is approximately 3000 miles from New York to San Francisco. So for two round trips, the total distance flown will be 12,000 miles. Converting to kilometers:

$$12000\,\text{mi} = 12000\,\text{mi}\left(\frac{5\,\text{km}}{3\,\text{mi}}\right) = 20000\,\text{km}\,. \tag{14.12}$$

We can now figure out the energy used using the fact that planes use 40 kWh to move one person 100 km:

$$20000\,\text{km} = 20000\,\text{km}\left(\frac{40\,\text{kWh}}{100\,\text{p}\,\text{km}}\right) = 8000\,\text{kWh}\,. \tag{14.13}$$

Dividing this by the number of days in the year to get the energy use per day, we find:

$$\frac{8000\,\text{kWh}}{365\,\text{d}} \approx 22\,\text{kWh/ d}\,. \tag{14.14}$$

This is a lot of energy—almost ten percent of the total 250 kWh/day used by the average American.

Here is another way to think about how much energy these two flights are. Let's take kWh/day and convert to kW:

$$22\,\text{kWh/ d} = \frac{22\,\text{kWh}}{1\,\text{d}}\left(\frac{1\,\text{d}}{24\,\text{h}}\right) \approx 0.92\,\text{kW}\,. \tag{14.15}$$

So, rounding up a bit, flying across North America twice per year corresponds to a power of one kW. One kilowatt is the power drawn by

Figure 14.4: Taking two transcontinental flights uses as much energy as leaving this toaster on non-stop for an entire year. (Image source: Donovan Govan, posted on wikipedia https://en.wikipedia.org/wiki/File:Toaster.jpg, licensed under the GNU Free Documentation License https://en.wikipedia.org/wiki/en:GNU_Free_Documentation_License.

a typical toaster. So taking these two round-trip flights uses as much energy as running a toaster 24 hours a day for an entire year.

How much carbon dioxide is produced by flying and what are its effects? It turns out that there is not quite universal agreement this question. To read more about where some of the disagreements lie, a good starting point is the article by Wihbey 2015, available at http://tinyurl.com/j4yclnd. Despite some a lack of unanimity about the details of how to account for the greenhouses gasses due to flying, the big picture is clear: Flying takes a lot of energy, this energy is produced from fossil fuels, and so flying is a large source of greenhouse gases. Estimates seem to be that flying is responsible for two to four percent of all anthropogenic climate change.[6]

[6] See http://www.grida.no/publications/other/ipcc_sr/?src=/climate/ipcc/aviation/index.htm and https://en.wikipedia.org/wiki/Environmental_impact_of_aviation#Total_climate_effects.

A little bit of Plane Physics

So planes are a lot like cars, except they fly through the air instead of roll along the highway.[7] But this flying business makes a difference in our basic model of transportation. Planes need to use a certain amount of energy just to stay up in the air. This is in addition to the energy they need to use to move forward. It turns out that there is no energetic advantage to flying slowly. If a plane goes faster it takes more energy to move it forward, but the plane gets more lift so less energy is needed to stay up. The velocity-dependence of these two things—lift and drag—tend to cancel out, so that the energy consumption of planes are essentially constant. What this means in practical terms is that we couldn't double the fuel efficiency of planes by flying half as fast.

[7] Add more later from MacKay Chapter C? Not sure how much we went to get into this. Or maybe just keep this to a paragraph.

This article: https://www.yaleclimateconnections.org/2015/09/evolving-climate-math-of-flying-vs-driving/ looks like a good overview of the climate and energy impacts of flying and driving.

14.6 Transporting Stuff

Energy intensity of different forms of shipping:[8]

- Road: 1 kWh per ton-km
- Container Ship: 0.015 kWh per ton-km
- Plane: 1.6 kWh per ton-km
- Rail: 0.1 kWh per ton-km

[8] I think shipping maybe should be its own chapter? Or maybe should be part of the Making Stuff chapter?

14.7 Conclusion

- Gasoline: 10 kWh per liter or 38 kWh per gallon
- Typical gas mileage for car: 30mph, but this ranges considerably.
- Flying uses roughly 40 kWh per 100 p-km.

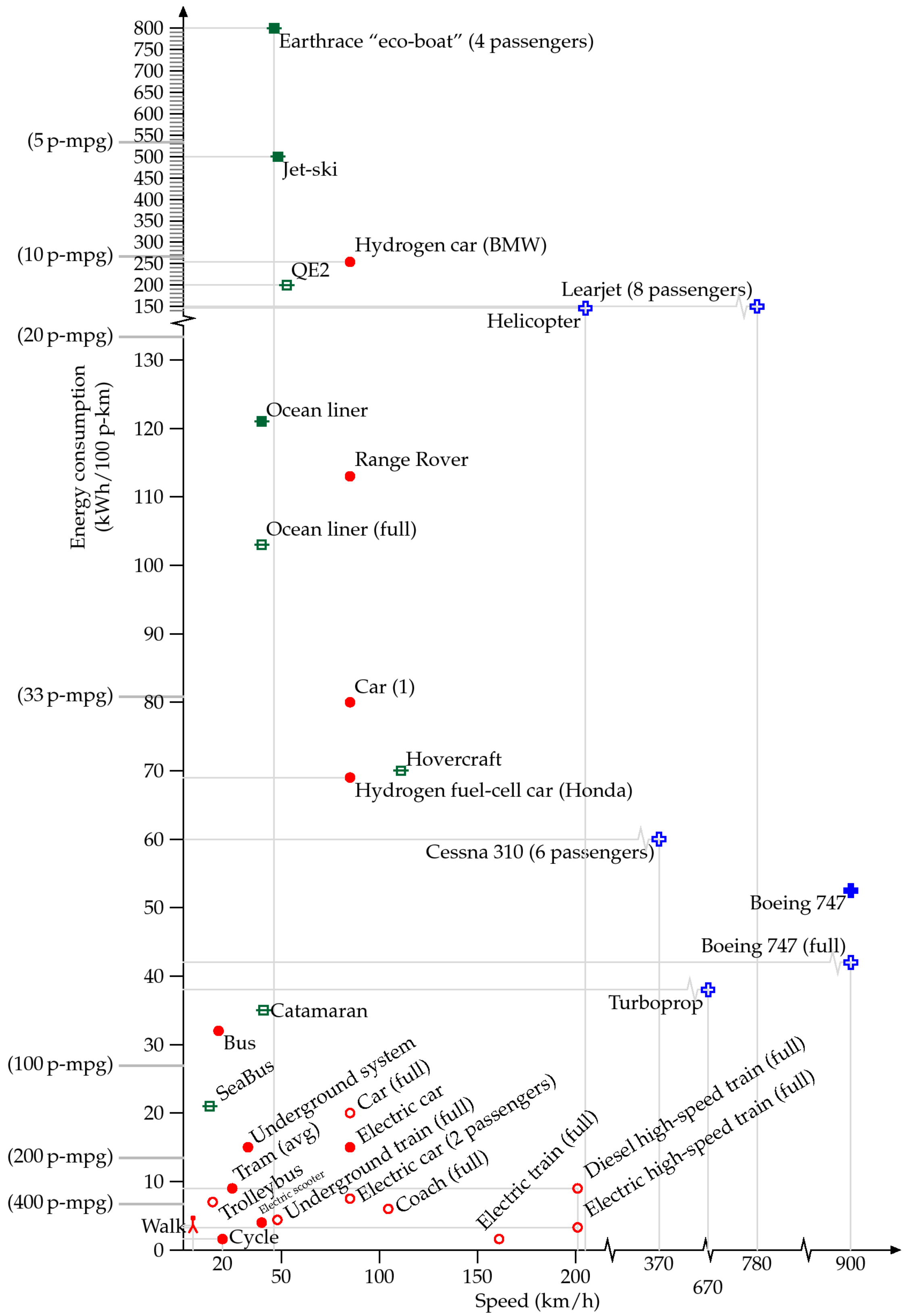

Figure 14.5: Figure 20.23 from MacKay (2009). Comparing the energy use and speed of different forms of transportation.

- A typical car (with one passenger) uses roughly 80 kWh per 100 p-km.
- An electric car uses roughly 15 kWh per 100 p-km.
- Carbon intensity of gasoline: 240g per kWh.
- Carbon intensity of electricity generation in the US: 613 g per kWh.

14.8 Exercises

Exercise *14.1:* Outside of the US, it is common to express fuel efficiencies in units of liters per 100-kilometer instead of miles per gallon. Convert 30 mpg to L/100-km.

Exercise *14.2:* The average US driver drives 13,500 miles a year.[9]

[9] https://www.fhwa.dot.gov/ohim/onh00/bar8.htm

1. How many kWh of energy does this use per day? Is this a little or a lot?
2. How much CO_2 is emitted into the atmosphere in one year as a result of driving this distance. Is this a little or a lot?

Exercise *14.3:* Suppose you make one round trip flight a year from New York to Beijing. About how much energy does this take? Is this a little or a lot?

Exercise *14.4:* Suppose it takes a certain amount of time, traveling at a certain speed, to go a certain distance via car. Now suppose you travel twice as fast.

1. What happens to the duration of your trip?
2. What happens to the power used by your car when you are driving?
3. What happens to the total energy used by your car to complete the trip?

Exercise *14.5:*

Suppose someone invents a super-strong plastic that can be used to make cars that have half the mass as traditional metal cars. Would this improve the power consumption of city driving, highway driving, or both? Briefly explain.

Figure 14.6: One Toyota Prius and two sea kayaks. Figure source: http://priuschat.com/threads/kayaks-2-roof-rack-or-trailer.24763/ (Replace with picture of Doreen's car with kayaks.)

Exercise *14.6:* You are driving from Mount Desert Island to Rivière-du-Loup, Québec, in a Toyota Prius that usually gets 45 mpg driving on highways. For this trip you have two sea kayaks attached to the roof of the car. Approximately what gas mileage do you think you will get under these conditions? (Briefly explain the reasoning behind your estimate.)

Exercise *14.7:* You need to go from Boston to Minneapolis to attend a Prince memorial concert. Approximately how much energy would you use if you:

1. Drove by yourself?
2. Took an airplane?
3. Took a train?

To answer these questions, refer to the figure on page 128 of MacKay's book, available at `http://withouthotair.com/c20/page_128.shtml`.

Exercise *14.8:* Let's see what happens if we plug some reasonable numbers in to Eq. (14.11). Use 70 miles per hour for the car's speed. At this speed, the air resistance term is much larger than the braking term. So ignore braking. Use 1 m^2 for the frontal area of the car, and 1.3 kg/m^3 for the density of the air.

1. Plugging in to Eq. (14.11), what do you get for the power used by a car?
2. If you drive for two hours, how much energy would the car use?
3. How does this answer compare to the energy use for a car that we found in Example 14.1?

Exercise *14.9:* In this problem you will investigate the transition from city to highway driving.

1. Show that if $m_c > \rho A d$, then the braking term is larger than the air resistance term in Eq. (14.11).
2. Using the same numbers that you did in Exercise 14.8 show that if the distance d between stops is less than approximately 750 meters, then the braking term in Eq. (14.11) is larger than the air resistance term.

Exercise *14.10:* Go to `https://flowcharts.llnl.gov/commodities/energy` and select the most recent energy flowchart for the US.

1. How much energy is used for transportation in the US in

one year? Express your answer in kWh/p/day and also in Giga Watts.

2. Suppose we wanted to use electricity for this energy. How much electric power would we need to do so? Assume that gas engines are 25% efficient and the electric engines are 90% efficient.

3. If we wanted to generate this amount of electricity using wind power, how much land would be needed? Express this area in a meaningful way

4. If we wanted to generate this amount of electricity using solar PV how much land would be needed? Express this area in a meaningful way.

5. If we wanted to generate this amount of electricity using nuclear power, how many 2-GW nuclear generating stations would we need to build?

15 Nuclear Power

15.1 Chemical Reactions

Before we look at nuclear reactions, let's talk some about chemical reactions. A chemical reaction is one in which chemical bonds are rearranged: molecules are typically broken apart and new molecules form, often releasing energy. A familiar example is combustion. As an example, here is the chemical reaction that occurs when one burns methane, the main component of natural bas:

$$CH_4 + 20_2 \;\rightarrow\; 2H_20 + CO_2 + \text{energy}\;. \tag{15.1}$$

The methane, CH_4 combines with oxygen and is converted to carbon dioxide, water vapor, and energy. Burning one kilogram of methane will produce around 50 MJ of thermal energy. This energy can be used to heat your house, cook food, or produce steam that can turn a turbine and generate electricity.[1]

As we know, some initial heat is needed to begin combustion.[2] Once started, the burning process continues on its own. Some of the heat released by the chemical reaction in Eq. (15.1) is used to combust other methane molecules. The 50 MJ per kilogram that we get out is the net energy released—the energy that does not go into starting other molecules burning.

Once the combustion process starts, it is important to know how to make it stop. Fortunately, we know three ways to put out fire. First, we can remove the fuel. If, for example, we are gradually releasing methane into a hot boiler, we can stop releasing methane and, after the remaining fuel has burned, the fire is over. Second, we could cut off the supply of oxygen. Third, we could remove heat from the fuel. When we put water on on fire, it typically has the effect of both denying oxygen and cooling off the fire so the burning stops.

[1] Consulting a number of different sources yields a range of values for the heat of combustion of methane. Why is this? Perhaps different assumptions are being made about temperature or pressure?

[2] You can't start a fire without a spark Springsteen (1984).

15.2 Nuclear Fission

In the next section we'll look at nuclear fission and compare and contrast it to a chemical reaction. In a chemical reaction, chemical bonds are rearranged; in a nuclear reaction, nuclear bonds are rearranged. However, there are some important differences between these processes that will be discussed below.

Consider the isotope of Uranium with 235 nucleons: the total number of neutron and protons is 235. This atom is called Uranium-235 and is denoted by ^{235}U. The atomic number of Uranium is 92. This means that it has 92 protons. It thus has $235 - 92 = 143$ neutrons.

Uranium-235 is only a small fraction of naturally occurring Uranium, in which for every ^{235}U atom there are 140

The nucleus of an atom is made up of protons and neutrons (except for hydrogen, whose nucleus consists of a single proton).

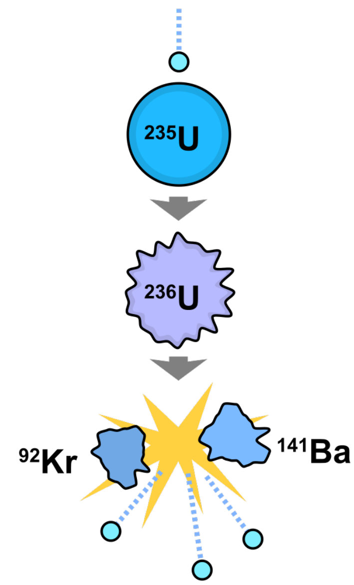

Figure 15.1: Nuclear fission that occurs when a ^{235}U atom under goes fission after absorbing a neutron. The large atom splits into two pieces, releasing energy and three additional neutrons. (Figure Source: Wikimedia Commons, released into the public domain by its author, Fastfission: https://en.wikipedia.org/wiki/File:Nuclear_fission.svg.)

15.3 Meltdowns and Accidents

15.4 Nuclear Waste

15.5 Nuclear Weapons and Terrorism

15.6 Exercises

Figure 15.3: The Seabrook Nuclear Power Plant Image source: Jim Richmond. Licensed under the Creative Commons Attribution-Share Alike 2.0 Generic license. https://creativecommons.org/licenses/by-sa/2.0/deed.en.

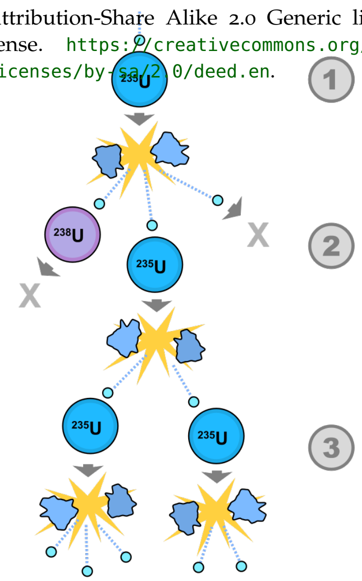

Figure 15.2: A schematic illustration of a nuclear chain reaction. Nuclear fission that occurs when a ^{235}U atom under goes fission after absorbing a neutron. The large atom splits into two pieces, releasing energy and three additional neutrons. (Figure Source: Wikimedia Commons, released into the public domain by its author, Fastfission. https://commons.wikimedia.org/wiki/File:Fission_chain_reaction.svg.)

Exercise *15.1:* The Seabrook Nuclear Power Plant, in Seabrook New Hampshire, is the second largest nuclear power plant in New

England. In 2016 it generated 10,800,000 MWh of electricity. The nameplate capacity of the plan is 1,240 MW.

1. What is the capacity factor of the plant?
2. How much area does the power plant take up? Answer this by finding the power station on Google maps.
3. What is the power density of the Seabrook plant. That is, how many Watts of electricity does it generate per square meter?
4. How much area would a solar PV installation need to be to produce as much power as the Seabrook plant? Express your area in a meaningful way.
5. The average home in New Hampshire uses 620 kWh of electricity per month. About how many homes could the Seabrook plant power?

16
The Energy of Making Things

16.1 Life Cycle Assessment

Let's break down the lifecycle of something into four phases.

1. **Raw materials.** The energy to get materials out of the ground and convert them to a useful form so we can use them as ingredients to make something.
2. **Production.** The energy required to build/make/assemble the thing.
3. **Use.** The energy associated with using a thing. For a car, which needs to be fed gasoline, this could be a lot. For a book, which doesn't need to be fed regularly in order to be used, this would be almost nothing.
4. **Disposal.** How much energy does it take to get rid of the thing? This could be a lot, if we recycle or need to process or dispose of the thing carefully. In a few cases it could be negative; if we get rid of a book by burning it, we could get a bit of energy back.

The total energy costs associated with all four phases of making an object is known as the *embedded energy* of that object. For example, the stainless steel bottle on the right weighs 0.32 pounds, or around 0.145 kilograms. LCA folks have determined that the embodied energy of stainless steel is 56.7 MJ per kg, or 15.75 kWh per kg. So, the embodied energy in the water bottle is:

$$0.145\,\text{kg}\left(\frac{15.75\,\text{kWh}}{1\,\text{kg}}\right) \approx 2.3\,\text{kWh}\,. \tag{16.1}$$

Figure 16.1: A stainless steel water bottle that will hold 18 fluid ounces of your favorite beverage. http://www.amazon.com/dp/B00SA2ZKKU/ref=twister_B00X6ZKQA0?_encoding=UTF8&psc=1

So it took around 2.3 kWh of energy to make the steel that was used to make the water bottle. The total embodied energy of the bottle would be higher, because we'd need to account for the energy used when taking the steel and converting it into something bottle-shaped. We'll also need to account for the energy needed to make the bottle purple, to make the lid, to ship all those materials around, and so on.

A few thoughts on LCA before launching into some examples.

1. LCA is limited by the quality of available data.
2. LCA give a number that represents an average. Thus, LCA ignores potentially large differences in the way that things are made. Of particular importance seems to be assumptions about whether or not ingredients are recycled or freshly-mined, requiring large energy costs to mine and process.
3. LCA reduces environmental impact to a single number, which may give one a false sense of simplicity.
4. On the other hand, it is surely very very important to think about the energy costs of making stuff, and LCA is probably the best place to start.

16.2 *LCA for Different Materials*

The LCAs for a handful of materials are shown in Fig. 16.1.

Material	Energy	Carbon
Stainless Steel	56.7	6.15
Steel	20.1	1.37
Polyurethane insulation (rigid foam)	101.5	3.48
Aluminum (general & incl 33% recycled)	155	8.24
Plywood	15	1.07
PVC	77.2	2.41
Iron	25	1.91
Glass	15	0.85

Table 16.1: Embodied energies and carbon for different materials. Energies are in units of MJ/kg. Carbon is in units of kg of CO_2 per kg. From the Circular ecology database, http://www.circularecology.com/embodied-energy-and-carbon-footprint-database.html, cited on https://en.wikipedia.org/wiki/Embodied_energy.

16.3 *LCA for Generating Electricity*

The results of an IPCC meta-study are shown in Table 16.2. Note the relatively large ranges for estimates of carbon intensity.

16.4 *LCA for Cars*

See the article by Xiaoyu Yan 2009. He assembles the results of a number of different studies that attempt to estimate the embodied energy and carbon of a car. The differences in estimations for the US are likely due to different assumptions about how much of the materials are recycled.

Table A.II.4 | Aggregated results of literature review of LCAs of GHG emissions from electricity generation technologies as displayed in Figure 9.8 (g CO_2eq/kWh).

Values	Bio-power	Solar		Geothermal Energy	Hydropower	Ocean Energy	Wind Energy	Nuclear Energy	Natural Gas	Oil	Coal
		PV	CSP								
Minimum	-633	5	7	6	0	2	2	1	290	510	675
25th percentile	360	29	14	20	3	6	8	8	422	722	877
50th percentile	18	46	22	45	4	8	12	16	469	840	1001
75th percentile	37	80	32	57	7	9	20	45	548	907	1130
Maximum	75	217	89	79	43	23	81	220	930	1170	1689
CCS min	-1368								65		98
CCS max	-594								245		396

Note: CCS = Carbon capture and storage, PV = Photovoltaic, CSP = Concentrating solar power.

Figure 16.2: The results of a meta-study of LCAs for different methods of generating electricity. From the IPCC Annex II Moomaw et al. (2011).

I think Yan seems to believe the larger estimate. The large GHG for China is due to the fact that much of their energy is derived from coal. The energy or carbon cost of making a car is a fairly significant fraction of its overall climate impact.

Country	Energy	Carbon
USA	119,150	10,480
USA	76,206	5511
USA	75,300	5390
Japan	61,500	5500
China	69,108	6575

Table 16.2: Different estimates of embodied energies and carbon for automobiles. Energy is in units of MJ; carbon is in units of kilograms of CO_2e. Data cited in Yan (2009).

Somewhere, either as an example or an exercise, consider whether it is good to buy a Prius right away or wait until your conventional car dies.

16.5 References

- `http://www.theguardian.com/environment/datablog/2011/apr/28/carbon-emissions-imports-exports-trade`.
- `http://www.pnas.org/content/107/12/5687.full`.

16.6 Exercises

Exercise *16.1:* Beverage containers..

1. Calculate the embodied energy and CO_2 of a 15 gram aluminum can.

2. Calculate the embodied energy and CO_2 of a 192 gram glass bottle.

Exercise *16.2:* A 2MW wind turbine requires around 80 tons of steel.

1. How much energy would such a turbine produce every month?
2. How much CO_2 is saved by the turbine, assuming that its electricity displaces electricity generated in the U.S?
3. What is the CO_2 cost of the steel in the materials?
4. What is its carbon payback time? That is, how long does the turbine have to run before the CO_2 savings from displacing traditionally generated electricity equals the CO_2 associated with the production of the steel used to make the turbine?

This is, of course, only a small part of the carbon dioxide associated with the raw materials and production of a wind turbine.

Exercise *16.3:* The Commonwealth Scientific and Industrial Research Organisation (CSIRO) in Australia has estimated that there are around 1000 GJ of embodied energy in the materials in a new house[1].

[1] http://www.yourhome.gov.au/materials/embodied-energy

1. Convert this energy into kWh.
2. If two people live in the house and the house lasts 100 years, how much energy is this in kWh per person per day?
3. Burning how many gallons of fuel oil would yield 1000 GJ?
4. How many year's worth of heating could you get from this amount of oil? Assume that the house uses 600 gallons of oil a year for heating.

Exercise *16.4:* Mike Berners-Lee [2] cites an estimate that the carbon cost of building a new, two-bedroom house is 80 tons. Let's round this up to 100 tons.

[2] Berners-Lee, Mike. How bad are bananas?: the carbon footprint of everything. Greystone Books, 2011.

1. Assume the house lasts for 100 years. How much carbon dioxide is this per year?
2. How much fuel oil, per year, would generate the same amount of carbon dioxide? Put this number in perspective.

17 Agriculture

This chapter is just some mostly unorganized notes right now.

17.1 How much Food do People Need?

Go through basic food example, discuss dietary calories versus actual calories. kcal to MJ example?

For measuring the energy/climate impacts of agriculture, we will use our usual units: tonnes of CO_2e per person per year or kWh per person per year. But it is less clear how to measure the benefit of agriculture? Mostly we'll think in terms of the kilograms of food produced or the kilocalories of food produced. Sometimes energy/climate impacts are expressed per dollar spent or person or household. Also, sometimes the impacts are expressed per acre or hectare. This maybe doesn't make the best sense, because what we're interested in is thinking about to feed people without frying the planet.

It's also worth noting that agricultural efficiencies can be measured in different ways. In general, we can think of the efficiency e as:

$$\text{Output} = \text{Efficiency} \times \text{Input} . \tag{17.1}$$

Just as there are different ways of measuring outputs (kcal, kg of food, dollar value of food), there are different ways of measuring inputs: area of land, amount of human labor, dollar value of all inputs, energy of all inputs, and so on. This is definitely something to be mindful of.[1]

[1] Cite Levins article(s). What else?

17.2 Overall Estimates of GHG associated with Agriculture

Agriculture is responsible for GHG emissions for two reasons: energy inputs and land-use impacts (LUC). Energy inputs include fertilizer, tilling soil, moving crops and fertilizer around, etc. Land-use impacts involves changes in C and N cycles as the result of bringing land in and out of production and changing agricultural practices. LUC is a complex topic that is beyond the scope of this book. But it is significant—likely at least as significant as energy inputs. Also, mention that energy inputs are broken down into primary and secondary.

- Pimentel (cited in (Pelletier et al. (2011))) estimated that the American diet is underpinned by 2000 liters of oil equivalents per year, accounting for 19% of the total US energy use.

- In terms of primary energy inputs, it seems that organic ag uses only a little bit less (perhaps around ten percent) than conventional ag. See Table 1 in Woods et al. (2010).

17.3 Transportation Costs

Data from Weber and Matthews (2008):

Mode	Energy	Carbon
rail	0.3	18
truck	2.7	180
air	10	680
container ship	0.2	14

Table 17.1: Energy and carbon dioxide associated with different modes of transportation. Energy is in units of MJ per ton-kilometer. Carbon is in units of tonnes of CO_2e per 10^6 ton-kilometers. Data taken from Table 1 of Weber and Matthews (2008).

17.4 Estimates of GHG Associated with Different Foods

From Weber and Matthews (2008), GHG emissions for different types of food, in units of kg of CO_2e per kilogram of food.[2]

[2] I eyeballed some of these of Fig. 2 from the Weber paper. Are the actual numbers published somewhere?

- Red Meat: 22

- Chicken/Fish/Eggs: 5.7

- Cereals/Carbs: 2.9

- Fruits/Vegetables: 2.2

- Oils/Sweets/Condiments: 2.2

A totally local diet would save 0.36 tCO_2e per year per household (Weber and Matthews, 2008).

This looks like a really good review article: `https://thebreakthrough.org/index.php/issues/the-future-of-food/the-future-of-meat`.

17.5 Why is Agriculture So GHG Intensive?

This is complicated and there are different answers for different sectors. Primary inputs can be large. These are things like diesel to plow fields and pumps to pump water. But the largest is probably nitrogen fertilizer.

Nitrogen

Plants need nitrogen to grow. Nitrogen is a constituent of most proteins and enzymes, and so plants need a source of nitrogen in order to make plant matter. They get this nitrogen from the soil. And so in order to maintain or increase yield, one very often has to add nitrogen to the soil. The significant (vast?) majority of nitrogen fertilizer is made via the Haber–Bosch process. This is an industrial process that takes nitrogen from the air (N_2), and turns it into ammonia (NH_3). Nitrogen is air is quite stable, so a lot of energy has to be added to the system to break it apart and form ammonia. The HB process is usually carried out at temperatures of $400 - 500$ degrees Celsius and pressures of $150 - 250$ atmospheres. It takes a lot of energy to maintain pressures this high, and producing nitrogen in this way is quite energy intensive. This translates into a lot of CO_2e. One kilogram of nitrogen fertilizer is responsible for the release of 6.69 kg of CO_2e (cited in Woods et al. (2010), Table 5).

Some estimate[3] that worldwide around five percent of all natural gas burned is used to make nitrogen fertilizer. Eighty percent of the N fertilizer in China is produced by coal.

[3] Need citation(s).

Nitrogen is about 5 (or more?) times more GHG intensive to produce than P or K fertilizer.

Ruminants

Another big issue when thinking about GHG and agriculture is the effect of ruminants. Ruminants are mammals that have a rumen—a funny sort of pre-stomach in which they kinda ferment their food before digesting it. This fermentation processes produces methane, which is a very powerful greenhouse gas. The large greenhouse gas emissions associated with "red" meat is in large part due to the methane emissions from the animals before they are eaten. Cows are the most famous and notorious of the ruminants, but sheep, goats, and deer are also ruminants.

Some facts quoted in Ripple et al. (2014):

- 14.5% of all anthropgenic emissions are from livestock sector.
- of this 14.5%, 44% is due to enteric CH_4.
- 27% is from land-use change and fossil-fuel use
- and 29% is from N_2O from manure and fertilizer applied to feed-crops

- Total area used for grazing cattle is 26% of the terrestrial surface of the earth.
- livestock sector accounts for 70% of the global agricultural land
- And 33% of the world's total arable land is used to grow feed-crop—grain that is fed to cattle. This is land that could be used directly to feed people, or to grow plants that could be used for generating electricity or fuel.

17.6 What is to be Done?

List based on Garnett 2011.

- Eat less meat and dairy, especially less red meat. (high priority)
- Eat less. (high priority)
- Reduce waste. (medium/high)
- Eat seasonal food and/or foods that store well. (medium)
- Accept variability of supply. I.e., you can't eat tomatoes or mangoes in Canada all year long. (medium)

Other thoughts: eat lower on the food chain, eat less processed foods, eat in season. We need to expand our notion of local further down the food-processing chain. That is, not only should be farm from which food comes be close to us, but the plant that provides materials and machines to the farm should be close to the farm, and so on. But even then, we should bare in mind that transport, in toto, is still a small part of the overall energy/GHG cost of agriculture.

17.7 Misc References

Some review papers Pelletier et al. (2011); Garnett (2011); Pimentel et al. (2008); Woods et al. (2010). See also the Eshel, et al paper in PNAS http://www.pnas.org/content/111/33/11996.

See also the "Beans for Beef paper" Harwatt et al. (2017). And Nijdam et al. (2012). https://link.springer.com/article/10.1007%2Fs10584-017-1969-1: "Substituting beans for beef as a contribution toward US climate change targets."

See the fun article, "LED salad and Jevon's paradox" by David Keith[4] Looks like some great info and a fun sample calculation about lettuce.

[4] http://www.keith.seas.harvard.edu/blog/led-salad-and-jevons-paradox.

Perhaps see also "What's the Climate Ipact of Your Diet?" and references therein. https://makewealthhistory.org/2016/05/06/whats-the-climate-impact-of-your-diet/.

Also http://www.pnas.org/content/113/15/4146.full. "Analysis and valuation of the health and climate change cobenefits of dietary change." from PNAS.

17.8 Exercises

Exercise *17.1:* The cost of shipping

1. How much energy does it take to ship 3 metric tons of corn from Iowa to Bar Harbor via truck? How many tons of carbon dioxide does this emit?
2. How much energy does it take to ship 3 metric tons of potatoes from Belfast, Ireland to Boston via container ship? How many tons of carbon dioxide does this emit?
3. How much energy does it take to ship 3 metric tons of bananas from Ecuador to Boston via plane? How many tons of carbon dioxide does this emit?

Exercise *17.2:* Swapping red meat for beans and rice...

1. The total beef consumption in the US in 2015 was 24.8 billion pounds.[5] Convert this to kilograms of beef per person per day, and kilograms of beef per person per year.
2. What is the per person CO_2e assoiated with this rate of beef consuption? Answer in tons per person per year. Put this number in perspective.
3. Suppose you replace this beef consumption with beans. How much CO_2 have you prevented from being emitted in one year? Put this number in perspective.
4. How much driving in an average car would emit an amount of CO_2 equivalent to the amount of CO_2e saved by this dietary switch?

[5] https://www.ers.usda.gov/topics/animal-products/cattle-beef/statistics-information.aspx, accessed October 30, 2017.

For more on this idea, see Harwatt et al. (2017)

Exercise *17.3:* David Pimentel and collaborators 2008 in a paper in the journal *Human Ecology* have estimated that the typical American requires the equivalent of 2000 liters of oil to supply their food.

1. How much energy is this in kWh/day?
2. How much CO_2 is this in tonnes per year?

18
Biofuels

This chapter is just some mostly unorganized notes right now.

The idea of biofuels is to grow plants—corn, sugar cane, beets, whatever—and then use the plants to meet some part of our energy needs. There are three main ways this is done:

1. Plants are converted to diesel or ethanol, which is then used to make cars and trucks go.

2. Plants are burned directly to create thermal energy to make our houses warm in the winter.

3. Plants are burned (or products derived from plants) to make electricity.

Of course another traditional thing to do with sugars from plants is to feed them to humans and/or animals.

18.1 Basic Considerations

The process through which plants turn carbon dioxide[1] and sunlight into energy is *photosynthesis*. How efficient can plants do this? The range seems to be between 3%–6% Ehrlich (2013). This is just the efficiency for converting sun energy into chemical energy in the plants. We'd then have to figure out how efficiently we can do other things with this chemical energy: turn it into ethanol, burn it to heat our homes, or turn it into electricity.

[1] Plants eat CO_2 from the atmosphere.

Part of the reason that this efficiency is so low is that plants make use only of light in a fairly narrow band of wavelengths. Also, efficiencies tend to drop in higher light levels. When it is very sunny, plants can harness only a fraction of the photons that hit it. A good overview of biochemical and biophysical limits biofuel efficiency is the article by Sinclair in *American Scientist* Sinclair (2009).

The basic efficiencies for biofuel are going to come out very low compared to photovoltaics, which produce very roughly 10 W/m^2.

However, one important thing that biofuels have going for them is that they are easy to store, whereas electricity is very difficult to store—doing so requires batteries of some sort which are expensive.

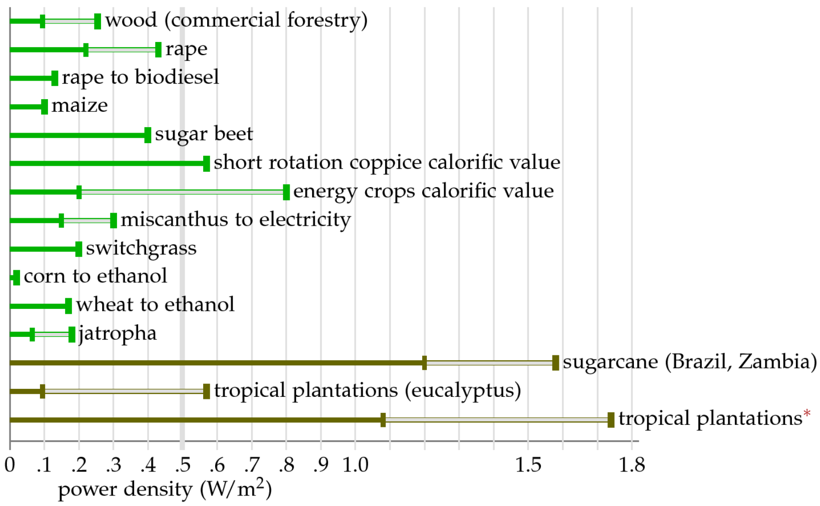

Figure 18.1: Efficiencies for various forms of bio-energy. Figure 6.11 from MacKay (2009).

Figure 18.1 (MacKay, 2009, Fig. 6.11) shows the efficiencies of various forms of bio-energy in units of W/m^2. All of these efficiencies are puny compared to solar PV. The statistics shown in green are for northern Europe. Based on this, it seems very unlikely that bioenergy could be a reasonable contributor to energy in northern Europe or similar climates. There is some hope that bioenergy grown in tropical regions could contribute non-negligibly.

18.2 Some Facts about Ethanol

One thing we can do with chemical energy (sugars) from plants is to turn them into ethanol. Doing so requires some processing which takes energy. Some ways to measure the efficiency of bioethanol:

- *Net energy ratio* or NER. This is the ratio of the energy supplied by a biofuel to the energy required to create it. For example, an NER of 1.5 means that to get 1.5 kWh of energy would require an input of 1 kWh.
- Fuel produced per unit area of land. This doesn't account for the energy used to process the plants into whatever form we want.
- GHG reduction. This measures how much less GHGs are produced by the biofuel compared to the comparable non-bio fuel. This usually[2] takes into account the changes in land use associated with the production of the plants.

[2] Is this standard?

The two biggest producers of bio-ethanol are the US and Brazil. But the two countries use different crops to do so, with very different results. In the US, we turn corn into ethanol; in Brazil they use sugar cane. Some facts (Ehrlich, 2013, p. 140):

- Brazil: 1798 gallons of ethanol per hectare. NER is $8.3 - -10.2$. GHG reduction is 61%.
- US: 900 gallons of ethanol per hectare. NER[3] is 1.3. GHG reduction is 19%.

[3] This seems high. Elsewhere I've seen estimates that are a bit lower.

Ethanol can be used in cars. However, the miles per gallon for ethanol is about 1/3 less than for gasoline. (I should check this. I think it's more like 40% less?)

18.3 Outlook

- Unlikely to increase efficiencies of plant photosynthesis, as they are already pushing against biophysical and biochemical limits.
- Biofuels probably have a role to play, but a small one, in a sustainable energy future. Biofuels can be used to replace fossil fuels when electrification isn't possible. A prime example is jet fuel.
- Corn ethanol subsidies in the US are insane. We spend a lot of money propping up a technology that is only marginally better from a carbon point of view than gasoline, while taking land out of food production. This is great for corn farmers, but not-so-great for the rest of us.
- Biofuels or bioenergy is probably not a useful term, as it is so broad. It encompasses a very wide range of stuff.
- There's some reason to believe that algae technology could be developed further and could be feasible. (But does this all depend on growing the algae in an enhanced-CO_2 environment?)

18.4 Misc References

18.5 Exercises

Exercise *18.1:* The average miles driven by a car annually in the US is $14,425$. In so doing the average car uses 733 gallons of fuel. There are $221,000,000$ cars in the US.[4]

1. What is the mpg of the average car in the US?
2. How much fuel is used in the US by all cars in one year?

[4] All figures from `https://www.fhwa.dot.gov/ohim/onh00/onh2p11.htm`. I think this data is from 1990.

3. Suppose we wanted to replace all of this gasoline with ethanol derived from US corn? How much land would be required? Express this area in some way that makes sense.

(Note, btw, that this doesn't include the fuel used for trucks and planes.)

Exercise *18.2:* To heat an average home in Maine requires approximately 540 gallons of fuel oil per year.[5]

1. How much thermal energy does this fuel oil produce? Answer in BTUs and kWh.
2. What power does this correspond to?
3. Suppose that you decide to heat your house with wood. Assume that the efficiency of your woodstove will be the same as the efficiency of your oil furnace.[6] How much land would you need to get this amount of power? Refer to Fig 18.1 from the textbook.
4. There are very roughly half a million homes in Maine[7] How much land would be needed if all of Maine was to heat with wood? Put this number in perspective. Is this a little or a lot? What fraction of Maine is this?

[5] https://www1.maine.gov/energy/fuel_prices/heating-calculator.php, accessed November 6, 2017.

[6] This might not be a super assumption, but since in what follows we're interested in getting a very rough estimate, I think this assumption is ok.

[7] http://www.census-charts.com/HF/Maine.html, accessed November 6, 2017.

Exercise *18.3:* This exercise is based on problem 4 on page 153 of Ehrlich (2013). Suppose a farmer has 10 pigs and wants to use the pigs' waste to generate some electricity. Assume that each pig generates 1 kg of solid waster per day, from which one can obtain 0.8 m^3 of methane. Burning one cubic meter of methane will produce 38 MJ of thermal energy.

Figure 18.2: A pig in a bucket. Ben Salter, licensed under the Creative Commons Attribution 2.0 Generic license, https://creativecommons.org/licenses/by/2.0/deed.en Available at https://commons.wikimedia.org/wiki/File:Pig_in_a_bucket.jpg.)

1. Let's suppose that we can turn the thermal energy from burning methane into electricity with a 25% efficiency. How much electric power would the farmer get? Express your answer in kW.
2. Put this amount of energy into perspective. Could this amount of power be sufficient to provide electricity to the farmer's home?

Exercise *18.4:* In the US, one gets around 900 gallons of corn ethanol from one hectare of land every year.

1. Convert this into Watts per square meter.
2. Put this number into perspective. How does it compare to the power density of solar PV and terrestrial wind turbines?

Exercise *18.5:* Repeat the above question, but for ethanol from Brazilian sugar cane, which produces around 1800 gallons of ethanol per hectare.

Figure 18.3: Sugarcane being harvested in São Paulo state, Brazil. Image source: https://commons.wikimedia.org/wiki/File:Caminh%C3%A3o_Carregado.jpg, released into the public domain by its author, Edrossini.

Exercise *18.6:* The total amount of jet fuel used by US airlines in 2016 was roughly 17 billion gallons.[8]

1. Suppose we want to replace this fuel with ethanol derived by corn. How much ethanol would be needed to do this. Note that the energy content of ethanol is about 1/3 less than that of gasoline.[9] Since ethanol has 2/3 the energy of gasoline (and jet fuel), we would need to replace every gallon of gasoline with 1.5 gallons of ethanol.
2. How much land in the US would be needed to produce this amount of ethanol from corn?
3. Put this area in perspective. Express it a meaningful way.

[8] https://www.eia.gov/todayinenergy/detail.php?id=31512, accessed November 6, 2017.

[9] NB: We did not account for this in class on Monday!

19
The Grid

19.1 The Grid

19.2 Overview

19.3 Feed-in Tarrifs

This article from NREL might be a good primer `http://www.nrel.gov/tech_deployment/state_local_governments/basics_tariffs.html`.

19.4 Smart Grids

Say what is meant by this. Why smart grids matter.

19.5 Smart Grids and Electric Vehicles

Another reference to check out `http://erpuk.org/project/managing-flexibility-of-the-electricity-sytem/`. Looks like

Example 19.1. *Statement of example*
Solution goes here

19.6 Batteries

Some battery facts. Batteries store energy. How much?

- AAA batteries store around 1.5 Wh of energy (or 0.0015 kWh).
- Prius (hybrids) have a battery capacity of around 1.5 kWh
- The powerwall by Tesla has a capacity of 13.5 kWh and costs around $6000, but this does not include installation. With installation, costs are $8,200-$14,200.[1]

[1] `https://www.energysage.com/solar/solar-energy-storage/tesla-powerwall-home-battery/`

- The second-generation Chevy Volt has an 18.4 kWh battery.

All facts are from wikipedia pages unless noted.

There are 250 million vehicles in the US. If they all had a 15 kWh battery, similar to the Chevy Volt, what is the total storage capacity? Let's see

$$250 \times 10^6 (15\,\mathrm{kWh}) \left(\frac{1\,\mathrm{GWh}}{10^6\,\mathrm{kWh}} \right) = 3750\,\mathrm{GWh}\,. \qquad (19.1)$$

I suspect that storage and batteries will eventually become their own chapter. Time will tell.

19.7 Exercises

Exercise *19.1:* The Raccoon mountain pumped storage facility, one of the largest in the US, can store around 35 GWh. For how long could this "battery" provide electrical power to all the homes in Maine?

20 Carbon Capture and Negative Emissions

blah blah

20.1 How to Capture Carbon

20.2 How to Store Carbon

Some notes to myself:

- See the power set of slides that I made for class
- Check out this article. It looks good: http://www.carbonbrief.org/in-depth-experts-assess-the-feasibility-of-negative-emissions.
- Maybe also http://www.carbonbrief.org/carbon-capture-essential-for-climate-friendly-fracking-in-uk-says-utm_content=buffer88eb2&utm_medium=social&utm_source=twitter.com&utm_campaign=buffer?
- This looks good: https://www.carbonbrief.org/beccs-the-story-of-climate-changes-saviour-technology
- And http://www.pnas.org/content/109/14/5185.full
- Another thing: https://makewealthhistory.org/2016/06/10/turning-waste-co2-into-stone/.
- http://www.carbonbrief.org/beccs-the-story-of-climate-changes-saviour-technology
- Biophysical and economic limits to negative CO_2 emissions. Article from Nature Climate Change in 2015: http://www.nature.com/nclimate/journal/vaop/ncurrent/full/nclimate2870.html
- Biofuels: http://www.climatecentral.org/news/study-biofuels-worse-for-climate-than-gasoline-20634.

20.3 Exercises

Exercise *20.1:* In this exercise you'll work through some calculations to get a sense of the scale needed for CCS. We're just interested in a rough estimate, so round to one or two significant digits throughout.

1. How much CO_2 is emitted per year by the US?
2. Suppose we want to put one tenth of this CO_2 underground. To do so, we would need to liquefy the CO_2. What is the volume of the CO_2 we would need to put under the earth? The density of liquid CO_2 is around 1100 kg per cubic meter.
3. Now suppose that we wanted to put the liquid CO_2 deep underground somewhere in a saline aquifer or old oil well. Let's imagine that this is an empty cavity that has a height of 100 meters. What would be the floor area of such a cavity sufficient to store this CO_2? Are you surprised by the answer?

Part IV

Appendices

A

Working with Numbers

A central goal of this text is to help you learn to think critically about numbers and to learn to present numerical information in a understanding and intellectually honest way. The numbers that arise in discuss and debates about energy and climate can be particularly vexing for several reasons. First, the numbers are often huge: teraJoules of energy, gigatons of CO_{2e}, megawatt hours of electrical energy, and quadrillions of BTUs. Second, the units on these numbers are often not well understood. Few people have a good feel units such as kWh, BTUs, or MW. Lastly, people often communicate these numbers in a way doesn't provide any context or frame of reference. Sometimes this is done intentially; large numbers can be used to intimidate or obscure. I think this happens on both "sides." Both environmentalists and anti-environmentalists have been known to present numbers in potentially misleading ways.

Here's a silly example. I am 1700 millimeters tall. Wow! That's a lot of millimeters. I am also 0.0017 kilometers tall. Wow! That's not a lot of kilometers. I can also express my height in a more conventional way; I am five feet, seven inches tall. This is about two inches shorter than the height of the average male in the US. So I'm a bit shorter than average.

This example is silly, because you are familiar with millimeters, kilometers, and the heights of men. So playing games like this seems daft and a bit puerile. But if you didn't know all this, you might be given the wrong impression by seemingly large or small numbers.

TODO! Use one or two actual examples where numbers are used in a misleading way?

This Appendix contains a number of guidelines and ideas for presenting numbers in a way that are understandable and intellectually honest.

A.1 Scientific Notation

We first need to talk about the scientific notation, which is a super useful way of representing very large or very small numbers. As an example, the population of the United States is: $325,000,000$. Read aloud, one would say "325 million". This is easier to understand than saying "three two five zero zero zero zero zero zero. Scientific notation is a way of numerically representing a number that dispenses with the need a long string of zeros. One would express the population of the US in scientific notation as follows:

$$325,000,000 \; = \; 3.25 \times 10^8 \; . \tag{A.1}$$

To see why this is true, recall that 10^8 is 10 times itself 8 times:

$$10^8 \; = 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \; = \; 100,000,000 \; . \tag{A.2}$$

Plugging this in to the right-hand side of Eq. (A.1), we have

$$3.25 \times 10^8 \; = \; 3.25 \times 10,000,000 \; = \; 325,000,000 \; . \tag{A.3}$$

A.2 Making Large Numbers Understandable

Two techniques:

- Divide a large number by the population size to arrive at a per capita quantity. We've done this when thinking about CO_{2e}emssions.
- Divide a large annual number by the number of days in a year. We've done this when thinking about energy consumption.

  Guiding principles

- Choose units and scales so that numbers are between 1 and 100 or perhaps 1000. I think most people have a good feel for numbers in this range.[1]
- Use units that are common and understood: miles, kilometers, pounds. Units such as kW and MWh are not commonly understood and so you will need to give you audience some sense of what these units mean.

Clearly and honestly communicating areas poses some additional challenges which are discussed in the following section.

A.3 Communicating Areas

A.4 Representing Numbers Visually

Some thoughts on good and bad or deceptive graphs and figures? Not sure if I need this section. Probably I don't.

[1] **TODO!** citations?

A.5 Estimation

Maybe this should be its own Appendix? This is how I feel now, but the future may be different.

A.6 Exercises

Exercise *A.1:* Convert the following numbers into scientific notation:

1. 1234
2. 430,000
3. 6,130,000
4. 14 trillion

Exercise *A.2:* In January 2016 the US national debt was 13.6 trillion dollars. Turn this into an understandable number by calculating the national debt per capita. Having done so, put this number into perspective, perhaps by comparing the per capita national debt to the amount of debt people sometimes go into to pay for college or a house.

Exercise *A.3:* The 2015 annual budget of the US National Endowment for the Arts is 146 million dollars. Turn this into an understandable number by expressing this number per capita. Put this number into perspective; is it large or small?

Exercise *A.4:* Convert one trillion seconds into an understandable number.

B
Unit Conversions

B.1 On the Importance of Developing a System

Talk about how good style is important. That you want to develop a solid, comfortable system for performing unit conversions. You might not need the full machinery of your systems for simple problems, although if you have a good system then you'll almost never make a mistake on simple problems. But when you really need a unit-conversion system—or, more generally, a systematic approach to solving problems—is on harder problems. You'll want to have a comfortable, familiar system in place before you get into more complex terrain.

B.2 Basic Conversions

I'll illustrate basic unit conversion with an example. Suppose you weigh 160 pounds and want to know how much that is in kilograms. The first thing we need to know is how kilograms and pounds are related:[1]

[1] A bunch of unit conversions are listed in Appendix D.

$$1\,\mathrm{kg} = 2.2\,\mathrm{lbs}\,. \tag{B.1}$$

We then use this fact to carry out the conversion:

$$160\,\mathrm{lbs}\left(\frac{1\,\mathrm{kg}}{2.2\,\mathrm{lbs}}\right) = 73\,\mathrm{kg}\,. \tag{B.2}$$

Let's dissect the above equation. We start with your weight of 160 pounds. You want to re-express this fact in kilograms. In so doing, we don't want to change the physical fact of your weight; we just want to express the same thing in a different way.

We know that we can multiply a number by 1 and the number is unchanged. In particular, we know that $160 \times 1 = 1$. The term in parentheses in Eq. (B.2) is just the number one in disguise. To see this, note the top and bottom of the fraction are the same. For example, $\frac{3}{3} = 1$, since 3 goes into 3 one time. Similarly, $\frac{1\,\mathrm{kg}}{2.2\,\mathrm{lbs}} = 1$, since 2.2

pounds go in one kilogram. So multiplying 160 kilograms by the term in parentheses doesn't change its value, since the parentheses term really just has a value of 1. So the result of Eq. (B.2) is to change units, but not the physical value of quantity.

A common issue when doing unit conversion is whether or not to divide or multiply. I.e., which term should go on top? Should it be $\frac{1\,\mathrm{kg}}{2.2\,\mathrm{lbs}}$ or , $\frac{2.2\,\mathrm{lbs}}{1\,\mathrm{kg}}$? We answer this question by looking at the units on all the terms in Eq. (B.2). We need pounds on the bottom to cancel the pounds that we started with attached to the 160. Then the units cancel:

$$160\,\cancel{\mathrm{lbs}}\left(\frac{1\,\mathrm{kg}}{2.2\,\cancel{\mathrm{lbs}}}\right) = 73\,\mathrm{kg}\,. \tag{B.3}$$

The strategy is to write out your conversion factor (the term in parentheses) with units attached. Choose which quantity goes on top so that the units cancel, leaving you with the new units that you desire. To make sure that you've done things correctly—it's super easy to insert an inversion factor upside down by mistake—make sure that the units cancel, as I've done in Eq. (B.3).

Let's try this technique out on a slightly more complex example.

Example B.1. *How many seconds are in one year?*
The key facts are: there are 365 days in a year, 24 hours in a day, 60 minutes in an hour, and 60 seconds in a minute. Putting this all together, we have:

$$1\,\mathrm{yr}\left(\frac{365\,\mathrm{d}}{1\,\mathrm{yr}}\right)\left(\frac{24\,\mathrm{h}}{1\,\mathrm{d}}\right)\left(\frac{60\,\mathrm{min}}{1\,\mathrm{h}}\right)\left(\frac{60\,\mathrm{s}}{1\,\mathrm{min}}\right) = 31536000\,\mathrm{s}\,. \tag{B.4}$$

Note that the units cancel, leaving us with seconds, as desired:

$$1\,\cancel{\mathrm{yr}}\left(\frac{365\,\cancel{\mathrm{d}}}{1\,\cancel{\mathrm{yr}}}\right)\left(\frac{24\,\cancel{\mathrm{h}}}{1\,\cancel{\mathrm{d}}}\right)\left(\frac{60\,\cancel{\mathrm{min}}}{1\,\cancel{\mathrm{h}}}\right)\left(\frac{60\,\mathrm{s}}{1\,\cancel{\mathrm{min}}}\right) = 31536000\,\mathrm{s}\,. \tag{B.5}$$

This large number of second is more conveniently expressed in scientific notation: $31536000\,\mathrm{s} = 3.15 \times 10^7$. Another way to say this is that there are around 32 million seconds in a year.

When I am assembling the conversion factors in Eq. (B.4), I say to myself, "There are 365 days in a year, there are 24 hours in a day," and so on. This makes it clear that what goes in each parenthetical factor is one in disguise; it's the same thing on the top and the bottom.

For all but the simplest conversions, I recommend writing out the conversion factors as I've done in Eq. (B.4). Some students learn to do unit conversions by slotting the conversion factors in a grid rather than parentheses. The two methods are, of course, equivalent. Use whichever makes you happier.

B.3 Converting Areas and Volumes

Converting areas and volumes follow the same general procedure as the conversion described above, but there is an added subtlety. I'll illustrate this with an example showing how to convert from square kilometers to square miles. First, a word about these square units. One square kilometer, 1 km^2, is an area[2] equal to the area of a square whose sides both have a length of one kilometer. A surface that has an area of 1 km^2 is not necessarily square in shape. It could be a triangle or a circle or a blob that happens to have an area the same as that of a square whose sides have a length of one kilometer. The unit 1 km^2 would be read aloud as either "one kilometer squared" or "one squared kilometer." Sometimes 1 km^2 is written as 1 sq km. I prefer 1 km^2, since it is easier to work with algebraically.

[2] **TODO!** make figure

Now for the example:

Example B.2. *The area of the US state of New Mexico is 315,000 km^2. What is the area of New Mexico in square miles?*

I'll use the fact that 5 km = 3.1 miles. Then,

$$315,000\,\text{km}^2\left(\frac{3.1\,\text{mi}}{5\,\text{km}}\right) = \text{Ummmm... err....No.} \tag{B.6}$$

Wait. This isn't going to work. There is a km^2 on top with the 315,000 but only a km downstairs with the 5 km. So the kilometers don't cancel: km^2/km $\neq 1$. We need another km downstairs. We can get one by squaring the conversion factor—the term in parentheses in Eq. (B.6). Can we just do that? Sure. The term in parentheses is just one in disguise. And $1^2 = 1$. So we can square one and still get one. So our unit conversion proceeds as follows:

$$315,000\,\text{km}^2\left(\frac{3.1\,\text{mi}}{5\,\text{km}}\right)^2 = 315,000\,\text{km}^2\left(\frac{3.1^2\,\text{mi}^2}{5^2\,\text{km}^2}\right) = 315,000\,\text{km}^2\left(\frac{9.61\,\text{mi}^2}{25\,\text{km}^2}\right) = 121,000\,\text{mi}^2\,. \tag{B.7}$$

Note that in the above equation *everything* inside the parentheses get squared: 3.1, 5, mi, and km.

Volume conversions proceed similarly to area conversions. Volume units are lengths cubed: meters^3, cm^3, and so on. In order to make the units work for volume conversions, we need to cube the conversion factor. I'll illustrate this with a short example

---

Example B.3. *A cube whose sides are* 10 *cm has a volume of one liter. How many cubic meters (m^3) equal one liter?*

The volume of cube of side s is equal to s^3. So

$$\text{Volume of one liter} = (10\,\text{cm})^3 = 10^3\,\text{cm}^3 = 1000\,\text{cm}^3\,. \tag{B.8}$$

Then, using the fact that 1 cm = 100 m

$$\text{Volume of one liter} = 1000\,\text{cm}^3\left(\frac{1\,\text{m}}{100\,\text{cm}}\right)^3 = = 1000\,\text{cm}^3\left(\frac{1^3\,\text{m}^3}{100^3\,\text{cm}^3}\right) = 0.001\,\text{m}^3\,. \tag{B.9}$$

Alas, this is not the end of the story. There are two other area units that are commonly used and which you should know about. We'll start with *acres*, which are an old farming unit that for some reason is still used today in the US, the United Kingdom, and some of the UK's former colonies. One acre is the area of a rectangle that is one chain (66 ft) by one furlong (660 ft) which happens to be equal to $1/640$ of a square mile. Ug. An acre also turns out to be 4047 m^2, which is a more useful fact for conversions.

How can one picture an acre? One acre is just about the size of one American football field, without the endzones. An acre is also a bit more than half of an full-size actual football (soccer) field. If you lived in medieval times, it might be useful to know that an acre is about how much land a team of oxen could plow in one day. Large forest fires in the US are often described with acres: usually tens or hundreds of thousands of acres. This seems completely daft to me; does anyone other than forest fire experts have a feel for what $10,000$ acres means? Acres are commonly used to express land areas in the US. For example, the land around my house that my spouse and I own happens to be around 2.47 acres. For this usage acres seem not unreasonable.[3]

[3] Well, there's still the unreasonableness of non-metric units, but in the US, that's sort of the way it goes.

Another special area unit is the *hectare*, defined as the area of a square with a side of 100 meters. So one hectare $= 10,000$ m^2. One hectare equals 2.47 acres—so by pure coincidence I happen to own one hectare of land. A hectare is about two and a half US football fields. The abbreviation for hectare is ha. Like acres, hectares are used commonly for measuring land areas, hectares being used in parts of the world where acres aren't.

It is important to remember that acres and hectares are already area units; they do not need to be squared. I'll illustrate this with an example.

Example B.4. *As I am writing this, there is a a wildfire in the Kenow Mountains in Alberta, Canada that has burned* $33,000$ *hectares. What is this area in square kilometers?*

$$33,000\,\text{ha}\left(\frac{10,000\,\text{m}^2}{1\,\text{ha}}\right)\left(\frac{1\,\text{km}}{1000\,\text{m}}\right)^2 = 33,000\,\text{ha}\left(\frac{10,000\,\text{m}^2}{1\,\text{ha}}\right)\left(\frac{1^2\,\text{km}^2}{1000^2\,\text{m}^2}\right) = 330\,\text{km}^2\,. \tag{B.10}$$

Note that the first conversion factor, 1 ha $=$ 1000 m^2, does not need to be squared. The second conversion factor, however, does need to be squared so that the units work out right.

Some strategies for visualizing and clearly communicating areas are presented in Section A.3. A handful of conversion factors for areas are included in Appendix D.

B.4 Exercises

Exercise *B.1:* Convert the following to meters

1. 5 miles
2. 1 kilometer
3. 1 inch
4. 100 yards
5. 3959 miles[4]
6. 6.2 miles

[4] The radius of the earth.

Exercise *B.2:* Some time conversions:

1. One billion seconds is how many years?
2. How many days in 80 years, a typical lifespan?

Exercise *B.3:* Convert the following quantities to kilometers

1. 6 miles
2. 100 yards
3. 60 miles

Exercise *B.4:* Convert the following speeds to meters per second

1. 100 km/hr
2. 55 mi/hr
3. 560 mi/hr[5]

[5] The cruising speed of a Boeing 787 aircraft (Boeing).

Exercise *B.5:* Convert the following to miles per hour:

1. 80 km/hr
2. 100 m/s
3. 3×10^8 m/s[6]

[6] The speed of light.

Exercise *B.6:* Look up the area of a football field.[7] The following areas are equivalent to how many football fields?

1. 1 acre
2. 1 hectare
3. 1 km^2
4. 1 mi^2

[7] Use either an American football field or a football football field—what we in America call soccer—as you prefer. But be sure to state which football field you're using.

Exercise *B.7:* The area of the state of Massachusetts is $10,565$ mi^2.

1. Convert this to square kilometers.
2. If Massachusetts was a square, how long would the side of the square be?

Figure B.1: Mount Desert Island, Maine, USA. Its shape reminds some of a walnut. Figure source: `https://commons.wikimedia.org/wiki/File:NPS_acadia-map.jpg`. Image produced by the US National Park service and edited by Matt Holly.

Exercise *B.8:* The area of Mount Desert Island (MDI) is 108 mi^2.

1. How many acres is MDI?
2. What is the area of MDI in square kilometers?
3. If MDI was a square, how long would the side of the square be?

Exercise *B.9:* The College of the Atlantic's campus is 37 acres. Convert this to:

1. Hectares
2. Square miles
3. Square kilometers

C
Proportional Reasoning

This probably doesn't need to be its own chapter. Likely it is better as a chapter in a larger appendix? Or maybe not. We'll see what develops.

In thinking about renewable energy, what often is most important is to know how two quantities change with each other. If we increase driving speed by ten percent, what would happen to the car's energy use (and hence fuel consumption)? If we double the length of a wind turbine's diameter, what happens to the energy generated by the turbine? Learning how to efficiently think through questions like these is know at *proportional reasoning*.

C.1 Proportionality

Quite often two quantities vary in proportion to each other. For example,

C.2 Not Every Relationship is Proportional

C.3 Exercises

Exercise *C.1:* Suppose you have a pizza of a certain area. If you triple the radius of the pizza, what happens to the pizza's area?

Exercise *C.2:* Suppose you have a pizza of a certain area. If you increase the radius of the pizza by ten percent, what happens to the pizza's area? What happens to the area if the radius is decreased by ten percent?

Exercise *C.3:* Which is more pizza: two pizzas of radius a or one pizza of radius $2a$? Explain.

D
Data

This is an appendix in which we can collect a bunch of data and conversions an other useful things. For now, let's just do lists. I can make pretty tables when the time comes.

D.1 Fundamental Definitions

- 1 A = 1 C/s
- $1e = -1.60 \times 10^{-19}$coulombs

D.2 Conversion Factors

D.2.1 Prefixes

- micro = μ = 10^{-6}
- milli = m = 10^{-3}
- kilo = k = 10^{3}
- mega = M[1] = 10^{6}
- giga = G = 10^{9}
- tera = T = 10^{12}
- peta = P = 10^{15}
- exa = E = 10^{18}

[1] Except for stupid BTUs, where M = 10^3 and MM = 10^6. See Section 5.2.

D.2.2 Lengths

- 5 km = 3.1 miles
- 1 ft = 12 inches
- 1 yard = 3 feet = 0.914 meters
- 1 mile = 5280 feet

D.2.3 Areas

- 1 acre = four rods $\times$ one furlong = 4047m^2
- 1 hectare (ha) = 10000 m^2 = 2.47 acres.

D.2.4 Volumes

- 1 gallons = 3.8 liters

D.2.5 Energy

- 1 kWh = 3,600,000 J = 3.6 MJ
- 1 kWh = 3412 BTU
- 1 BTU = 1055 J
- 1 dietary calorie = 1000 calories
- 1 calorie = 4.184 Joules
- 1 MBTU = 1000 BTU
- 1 MMBTU = 1,000,000 BTU
- 1 therm = 100,000 BTU
- 1 Quad = 10^{15} BTU
- 1 Quad = 293×10^9 kWh

D.3 Carbon Intensities and Other Data

D.3.1 Electricity Generation

Below is a list of the lifecycle carbon intensity of different forms of electricity generation. These numbers are averages, taking into account many different studies. Compiled from Moomaw et al. (2011), available at: `http://srren.ipcc-wg3.de/report/IPCC_SRREN_Annex_II.pdf`. All figures are expressed in gCO_2e/kWh

- Solar photovoltaic 46
- Concentrating solar power 22
- Geothermal 45
- Hydropower 4
- Ocean Energy 8
- Wind 12
- Nuclear Energy 16
- Natural Gas 469
- Oil 840
- Coal 1001

(Note to self: I might also want to pull data from the report available on the IEA webiste here: `http://goo.gl/yPBQdu`. It's a bit curious that the IEA's numbers are a bit different from the IPCCs. That said, the differences don't appear to be significant. (For example, the IEA has 400 for the carbon intensity of natural gas.) Check to see if the IEA uses CO_2or CO_{2e}.)

Average carbon intensity of electricity production by region in 2014.[2] Units: grams of CO_2/kWh.

[2] *CO_2 emissions from fuel combustion: highlights*. International Energy Agency. 2016.

- Worldwide: 521
- OECD Americas (Canada, Mexico, Chile, US): 441
- OECD Europe: 311
- Africa: 615
- Asia, excluding China: 685
- China: 681
- G20: 529

Note that in the OECD Americas the emissions are almost exactly one pound per kWh.

D.3.2 Calorific Value of Fuels

Calorific values of fuels. This is amount of thermal energy that results if one burns a kilogram or a liter of the fuel.

- Gasoline: 13.0 kWh/kg, 34.7 MJ/L, 120,480 BTU/gallon
- Coal: 8.0 kWh/kg, 19,100,000 BTU/short ton[3].
- Ethanol: 84,000 BTU/gallon
- Propane: 13.8 kWh/kg, 25.4 MJ/L, 91,600 BTU/gallon
- Natural gas: 14.85 kWh/kg, 0.04 MJ/L, 1,037 BTU/ft^3
- Heating oil: 12.8 kWh/kg, 37.3 MJ/L, 139,000 BTU/gallon
- Kerosene: 12.8 kWh/kg, 37 MJ/L, 135,000 BTU/gallon
- Wood: ∼4–5 kWh/kg
- Hardwood (maple): 24,000,000BTU/cord
- Softwood (norway pine): 17,000,000 BTU/cord

[3] A short ton is 2000 pounds

The BTU data can be found at https://www.eia.gov/energyexplained/index.cfm?page=about_energy_units. Lots of firewood data can be found at http://worldforestindustries.com/forest-biofuel/firewood/firewood-btu-ratings/

D.3.3 Carbon Intensity of Fuels

Carbon intensity of fuels (gCO_2/kWh). This this the amount of CO_2released if one burns a sufficient quantity of fuel to release 1 kWh of thermal energy.

- Natural gas: 190
- Propane: 217
- Gasoline: 240
- Diesel: 250
- Fuel oil: 260
- Coal: 300

D.3.4 Cost of Fuels

Fuel costs are notoriously volatile and fluctuate considerably from year to year and even from month to month. They also vary widely in different parts of the world. Current average fuel prices for Maine can be found at http://www.maine.gov/energy/fuel_prices/. The US Energy Information Administration also tracks weekly fuel prices at: https://www.eia.gov/dnav/pet/pet_pri_wfr_dcus_nus_w.htm

At the present moment[4], average prices in Maine are:

[4] September 2017

- Natural gas: $1.31/therm
- Heating oil: $2.03/gallon
- Wood pellets: $261/ton
- Kerosene: $2.56/gallon
- Propane: $2.36/gallon
- Cord wood: $250/cord

Having just looked up this data, I'm a bit troubled by the fact that I'm currently paying $3.70/gallon for propane. This might be something to look into. The wood price, however, is what I've paid per cord for the last few years.

D.4 Other Facts

- in the U.S., on average, one kWh of electricity production results in 613 grams of CO_2 being released into the atmosphere. The values for other countries can be found on page 335 of SEWTHA.[5]

[5] What is the original source for this? It's not clear. Hmm.... This page from the IEA appears to have a different number: https://goo.gl/JkjUJo

D.5 Some Useful Resources

- UNDP Human Development Reports. Tons of country-level data here, can be displayed in tables or downloaded in a well formatted spreadsheet. http://hdr.undp.org/en/data. Even more amazing is the UNDP's public data explorer: http://goo.gl/M9CvEQ. You can make graphs and animations of all UNDP data.

- The CAIT Climate Data Explorer http://cait.wri.org seems to be one of the most user-friendly places to get climate data, country-level GHG emissions, historical emissions, and so on. CAIT is a project of the World Resources Institute.

- The US Energy Information Administration http://www.eia.gov is another good resource for data. Has state-level info on consumption.

- The Lawrence Livermore National Laboratory makes some super-fun and interesting energy flow charge. You can access them at https://flowcharts.llnl.gov/commodities/energy.

Bibliography

Boeing. Boeing 787-8 Dreamliner Fact Sheet. URL http://www.boeing.com/boeing/commercial/787family/787-8prod.page. Accessed March 22, 2015.

Organisation Intergouvementale de la Convention du Mètre. The International System of Units (SI). Technical report, 2006.

Robert Ehrlich. *Renewable Energy: A First Course*. CRC Press, 2013.

Richard P. Feynman, Robert B. Leighton, and Matthew Sands. *The Feynman Lectures on Physics, Vol. 1: Mainly Mechanics, Radiation, and Heat*. Addison Wesley, 1 edition, February 1977. ISBN 0201021161. URL http://www.worldcat.org/isbn/0201021161.

Tara Garnett. Where are the best opportunities for reducing greenhouse gas emissions in the food system (including the food chain)? *Food Policy*, 36:S23–S32, January 2011. ISSN 03069192. doi: 10.1016/j.foodpol.2010.10.010. URL http://dx.doi.org/10.1016/j.foodpol.2010.10.010.

Helen Harwatt, Joan Sabaté, Gidon Eshel, Sam Soret, and William Ripple. Substituting beans for beef as a contribution toward US climate change targets. *Climatic Change*, pages 1–10, 2017.

David J. C. MacKay. *Sustainable Energy - Without the Hot Air*. UIT Cambridge Ltd., 1 edition, February 2009. ISBN 0954452933. URL http://www.worldcat.org/isbn/0954452933.

W. Moomaw, P. Burgherr, G. Heat, M. Lenzen, J. Nyboer, and A. Verbgruggen. Annex II: Methodology. In IPCC Special Report on Renewable Energy Sources and Climate Change Mitigation. *Intergovernmental Panel on Climate Change*, page 982, 2011.

Thomas Moore. *Six Ideas That Shaped Physics: Unit C: Conservation Laws Constrain Interactions*. McGraw-Hill Science/Engineering/Math, 2 edition, June 2002. ISBN 0072291524. URL http://www.worldcat.org/isbn/0072291524.

Durk Nijdam, Trudy Rood, and Henk Westhoek. The price of protein: Review of land use and carbon footprints from life cycle assessments of animal food products and their substitutes. *Food policy*, 37(6): 760–770, 2012.

Gian A. Pagnoni and Stephen Roche. *The Renaissance of Renewable Energy*. Cambridge University Press, 2015.

Nathan Pelletier, Eric Audsley, Sonja Brodt, Tara Garnett, Patrik Henriksson, Alissa Kendall, Klaas J. Kramer, David Murphy, Thomas Nemecek, and Max Troell. Energy Intensity of Agriculture and Food Systems. *Annual Review of Environment and Resources*, 36(1): 223–246, 2011. doi: 10.1146/annurev-environ-081710-161014. URL http://dx.doi.org/10.1146/annurev-environ-081710-161014.

David Pimentel, Sean Williamson, CourtneyE Alexander, Omar Gonzalez-Pagan, Caitlin Kontak, and StevenE Mulkey. Reducing Energy Inputs in the US Food System. *Human Ecology*, 36 (4):459–471, 2008. doi: 10.1007/s10745-008-9184-3. URL http://dx.doi.org/10.1007/s10745-008-9184-3.

Burton Richter. *Beyond Smoke and Mirrors: Climate Change and Energy in the 21st Century (Canto Classics)*. Cambridge University Press, 2 edition, December 2014. ISBN 1107673720. URL http://www.worldcat.org/isbn/1107673720.

William J. Ripple, Pete Smith, Helmut Haberl, Stephen A. Montzka, Clive McAlpine, and Douglas H. Boucher. Ruminants, climate change and climate policy. *Nature Climate Change*, 4(1):2–5, 2014.

UK National Health Service. What should my daily intake of calories be? http://www.nhs.uk/chq/pages/1126.aspx?categoryid=51. Accessed March 22, 2015.

William Shockley and Hans J. Queisser. Detailed Balance Limit of Efficiency of pâĂŘn Junction Solar Cells. *Journal of Applied Physics*, 32(3):510–519, March 1961. ISSN 0021-8979. doi: 10.1063/1.1736034. URL http://dx.doi.org/10.1063/1.1736034.

Thomas R. Sinclair. Taking Measure of Biofuel Limits: When pinning hopes on biofuels, an energy-hungry world must adapt to plant production capacities and resource limits. *American Scientist*, 97(5): 400–407, 2009. URL http://www.jstor.org/stable/27859392.

Susan Solomon, Dahe Qin, Martin Manning, Z. Chen, M. Marquis, K. Averyt, M. Tignor, and Miller, editors. *Climate Change 2007—The Physical Science Basis*. Cambridge University Press, 2007.

Benjamin K. Sovacool. Contextualizing avian mortality: A preliminary appraisal of bird and bat fatalities from wind, fossil-fuel, and nuclear electricity. *Energy Policy*, 37(6):2241–2248, June 2009. ISSN 03014215. doi: 10.1016/j.enpol.2009.02.011. URL http://dx.doi.org/10.1016/j.enpol.2009.02.011.

Benjamin K. Sovacool. The avian benefits of wind energy: A 2009 update. *Renewable Energy*, 49:19–24, January 2013. ISSN 09601481. doi: 10.1016/j.renene.2012.01.074. URL http://dx.doi.org/10.1016/j.renene.2012.01.074.

Bruce Springsteen. Dancing in the Dark. In *Born in the U.S.A.* Columbia, 1984.

phantom. US Department of Housing & Urban Development. 2015 Characteristics of New Housing. \protect\T1\textbraceleft https://www.census.gov/construction/chars/pdf/c25ann2015.pdf}, 2015.

Christopher L. Weber and H. Scott Matthews. Food-Miles and the Relative Climate Impacts of Food Choices in the United States. *Environ. Sci. Technol.*, 42(10):3508–3513, May 2008. doi: 10.1021/es702969f. URL http://dx.doi.org/10.1021/es702969f.

John Wihbey. Fly or drive? Parsing the evolving climate math. url{http://www.yaleclimateconnections.org/2015/09/evolving-climate-math-of-flying-vs-driving}/, accessed April 18, 2016., September 2015.

Jeremy Woods, Adrian Williams, John K. Hughes, Mairi Black, and Richard Murphy. Energy and the food system. *Philosophical Transactions of the Royal Society of London B: Biological Sciences*, 365(1554):2991–3006, September 2010. ISSN 1471-2970. doi: 10.1098/rstb.2010.0172. URL http://dx.doi.org/10.1098/rstb.2010.0172.

Xiaoyu Yan. Energy demand and greenhouse gas emissions during the production of a passenger car in China. *Energy Conversion and Management*, 50(12):2964–2966, December 2009. ISSN 01968904. doi: 10.1016/j.enconman.2009.07.014. URL http://dx.doi.org/10.1016/j.enconman.2009.07.014.