The SIR Model: Adding births and deaths

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Below is the SIR model with demographics. In this version we assume that the birth rate equals the death rate. We will denote this rate by μ. Note that μ is not death due to the disease; it's just the normal death rate for the population.

$$\frac{dS}{dt} = \mu - \beta SI - \mu S \,, \tag{1}$$

$$\frac{dI}{dt} = +\beta SI - \gamma I - \mu I \,, \tag{2}$$

$$\frac{dR}{dt} = +\gamma I - \mu R \,. \tag{3}$$

1. Modify the basic SIR code so that it incorporates demographics, as given by the above equations. This will likely take a little bit of work. In the R project I share with you there is a file called `SIR_without_demographics`. Don't edit this file. Make a copy called `SIR_with_demographics`, and work on that.

2. Does your demographics code work? Test it by setting $\mu = 0$. It should give you the same thing as when you run the `SIR_without_demographics` code (with the same parameters).

3. Ok. Now let's explore the model with demographics. Let's use the following assumptions:

   - $\beta = 2$.
   - People are infectious for an average of 7 days.
   - People live an average of 70 years.

   Note the different time units.

   (a) Enter the parameters in to your model, run it, and look at the results. It should be not very surprising.

   (b) Now run the model for $20,000$ days. That's not a typo. What do you observe? It should be rather surprising. It will be easiest to see what's going on if you only plot the infecteds and experiment with `ylim` and `xlim`.

   (c) What's going on epidemiologically? How can you make sense of your results?