The SIR Model with Vaccination

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Scenario 1: An Outbreak Entering a Vaccinated Population

Below is the SIR model. In this version we will ignore birth rate and death rate assuming that these are negligible in an outbreak. We will explore the vaccination level in the population necessary to prevent an outbreak. Instead we assume that a certain proportion of the population, V_0, is already vaccinated at the start of an outbreak giving us and initial susceptible population of $S_0 = S - I_0 - V_0$.

$$\frac{dS}{dt} = -\beta SI \ , \tag{1}$$

$$\frac{dI}{dt} = +\beta SI - \gamma I \ , \tag{2}$$

$$\frac{dR}{dt} = +\gamma I \ . \tag{3}$$

According to theory, an outbreak will be prevented in the effective R0 is less than 1. Choose different values for β and using the code calculate R0 and the critical proportion that needs to be vaccinated. Does the theory presented in the slides hold true?
The critical proportion to immunize is given by

$$P_{\text{crit}} = 1 - \frac{1}{R_0} \tag{4}$$

Scenario 2: Ongoing Pediatric Vaccination

For many human infectious diseases (e.g. measles, mumps, rubella, polio) there is an emphasis on vaccination of newborns or very young infants. Below the parameter p is used to denote the fraction of newborns (or infants) who are successfully vaccinated and as Dave would say "born again" into the immune class (in this case our recovered class). This term, p, is the product of the actual vaccination *coverage* (the percentage of newborns who receive the required number of vaccine doses) and the vaccine *efficacy* (the probability that they successfully develop immunity). Incorporated in the SIR model we get the following set of modified equations:

$$\frac{dS}{dt} = \nu(1-p) - \beta SI \ - \mu S \ , \tag{5}$$

$$\frac{dI}{dt} = +\beta SI - \gamma I - \mu I \ , \tag{6}$$

$$\frac{dR}{dt} = +\gamma I + \nu p - \mu R \ . \tag{7}$$

where ν is the birth rate, p is the product of the vaccination coverage and vaccine efficacy.
These equations are laid out in the script posted on google classroom to explore.