The SIR Model with demography: Adding waning immunity

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Below is the SIR model with demographics. In this version we assume that the birth rate equals the death rate. We will denote this rate by μ. Note that μ is not death due to the disease; it's just the normal death rate for the population.
In many diseases we have waning immunity, which can be incorporated into our models as follows:

$$\frac{dS}{dt} = \mu + wR - \beta SI - \mu S \,, \tag{1}$$

$$\frac{dI}{dt} = +\beta SI - \gamma I - \mu I \,, \tag{2}$$

$$\frac{dR}{dt} = +\gamma I - wR - \mu R \,. \tag{3}$$

Where w is the rate at which immunity is lost and recovered individuals move into the susceptible class.

1. Modify the basic SIR code with demography so that it incorporates waning immunity, as given by the above equations. This will likely take a little bit of work. In the R project I share with you there is a file called `SIR_with_waning_immunity`.

2. Does your waning immunity code work? Test it by setting $w = 0$. It should give you the same thing as when you run the code without w which is the equivalent of an SIR model with demographics.

3. Ok. Now let's explore the model with waning. Let's use the following assumptions:

   - $\beta = 2$.
   - People are infectious for an average of 7 days.
   - People live an average of 70 years.
   - People lose their immunity after one year (365 days) on average.

   Note the different time units.

   (a) Enter the parameters in to your model, run it for 100 days, and look at the result. It should be not very surprising.

   (b) Now run the model for $10,000$ days. What do you observe? Experiment with varying xlim and ylim to investigate the time series.

   (c) What's going on epidemiologically? How can you make sense of your results?

4. What happens as the waning immunity changes? Try the following.

   - People lose their immunity after 10 years.
   - People lose their immunity after 1 day.
   - People lose their immunity after 1 hour.