Homework Two

Introduction to Epidemiological Modeling

College of the Atlantic

Due Friday, April 14, 2023

Here are some instructions for how to submit this part of the assignment.

- Do the problems by hand using pencil (or pen) and paper. There is no need to type this assignment.
- If you like working on a tablet, go for it.
- Make a pdf scan of your work using genius scan or some similar scanning app. Please make the homework into a single pdf, not multiple pdfs or pngs.
- Submit the assignment on google classroom. Please don't email it to me. Thanks.
- If you want, you can do these problems in pairs and submit one assignment for the two of you.

1. A freshly-brewed cup of coffee is at 90°C. It sits on the table in a classroom in which the temperature is 20°C. What will happen to the temperature of the coffee? Make a qualitatively accurate sketch of the temperature $T(t)$ as a function of the time t, where t is measured in minutes. Think about the concavity of the graph.

2. The following differential equation is known as *Newton's Law of Cooling*:

$$\frac{dT}{dt} = -0.1(T - 20) \, . \tag{1}$$

   (a) Sketch the right-hand side of Eq.(1).

   (b) Sketch the solution to Eq. (1) for the case where $T(0) = 90$.

3. In its more general form, Newton's Law of Cooling is:

$$\frac{dT}{dt} = -k(T - A) \, . \tag{2}$$

   (a) What is the physical meaning of k? What happens to solutions of the differential equation if k is doubled?

   (b) What is the physical meaning of A? What happens to solutions of the differential equation if A is doubled?

4. Consider again Eq. (1) with $T(0) = 90$. Use Euler's method with $\Delta t = 5$ to come up with an approximate solution to the differential equation. Find a solution up to and including $t = 25$. Please show enough work so it's clear what you're doing.

5. It is possible to determine an analytic (I.e. an exact formula) solution to Eq. (2). The solution is:

$$T(t) = A + (T(0) - A)e^{-kt} . \tag{3}$$

Use calculus and algebra verify that the $T(t)$ of Eq. (3) is a solution to Eq. (2).

6. Use R to plot (on the same axes) your Euler solution from Problem 4 and the exact solution, Eq. (3), with $A = 20$ and $T(0) = 90$.

7. Use the Euler function in R to plot (on the same axes) the exact solution and Euler solutions for $\Delta T = 5$, $\Delta T = 1$, and $\Delta T = 0.1$.