Homework Three: SIR Explorations
Introduction to Epidemiological Modeling
College of the Atlantic
Due Friday, April 21, 2023

Here are some instructions for how to submit this part of the assignment.

- Do the problems by hand using pencil (or pen) and paper. There is no need to type this assignment.
- If you like working on a tablet, go for it.
- Make a pdf scan of your work using genius scan or some similar scanning app. Please make the homework into a single pdf, not multiple pdfs or pngs.
- Submit the assignment on google classroom. Please don't email it to me. Thanks.
- If you want, you can do these problems in pairs and submit one assignment for the two of you.

In class on Tuesday (18 April) we thought a bunch about the SIR model and then looked over code that solves the model and produces plots. I shared a program with you called `SIR Model and R's Built-in ODE Solver`. I recommend making a copy of this code so you have it to use as a starting point for subsequent assignments.

0. Experiment with the code a little. Change the values of β and γ and see what happens. To make the plots useful, you might need to adjust `times`. There is nothing to hand in for this question.

1. Suppose you set your model so that initially $I_0 = 1$ (and thus $S_0 = 0$). (Use $\beta = 1.5$ and $\gamma = 1/8$.)
   (a) What does this correspond to epidemiologically?
   (b) What does the SIR equation for I reduce to in this case?
   (c) Write down the solution to the I equation. I.e., write down[1] the formula for $I(t)$?
   (d) Use R to make a plot[2] of $I(t)$ do this two ways.
      i. Plot the numerical solution generated by `ode`.
      ii. Plot the exact solution that you wrote down in the previous problem.
   (e) **Optional:** What is the half-life of $I(t)$. Is the half-life you calculated consistent with what you see on your plots?

[1] You don't need to solve for anything. And you certainly don't need to do an integral. Just write down the solution. ☺

[2] Include the plots when you hand in the solution.

2. In this problem we'll focus on the $I(t)$ curve at the very beginning of an epidemic. Let's continue to use the parameters $\beta = 1.5$ and $\gamma = 1/8$. And let's go back to $I_0 = 0.001$.

   (a) Make a plot of $I(t)$ for the early days of the epidemic. For these parameters this will be around five days. What does the graph of $I(t)$ look like? There's no need to include a plot as part of your answer—you can just sketch it.

   (b) Let's think about why, mathematically, $I(t)$ looks the way it does. The $I(t)$ equation is:

$$\frac{dI}{dt} = \beta SI - \gamma I = I(\beta S - \gamma) \ . \tag{1}$$

   (c) Under what conditions will dI/dt be positive (and hence the disease will spread) at $t = 0$? Express your answer first as an equation among S, β, and γ. Then express your result as a sentence or two in English that explains this condition in practical terms.

   (d) We will think more about this condition and what it means practially on Friday.