An Overview of Spatial Epidemic Models

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Space!

- Epidemics unfold in space as well as time.
- So far we have not considered this.
- SIR models assume a single, well-mixed population.
- Often this is a good approximation. But sometimes it's not.

Figure: Van Gogh's The Starry Night coronavirus pandemic remix banner.

https://www.rawpixel.com/image/2320223/free-illustration-image-van-gogh-painting-art

Space!

- There are a number of different ways to incorporate a spatial component into a model.
- Some of these models are more spatial than others.
- Which to use depends on the situation you are modeling and the goal(s) of your model.

Figure: The Great Wave off Kanagawa against coronavirus background.

https://www.rawpixel.com/image/2324277/free-illustration-image-sea-waves-great-wave.

- A metapopulation is a distinct subpopulation.
- Metapopulations interact weakly.
- Example: on MDI, metapopulations might include COA, MDIHS, JaX

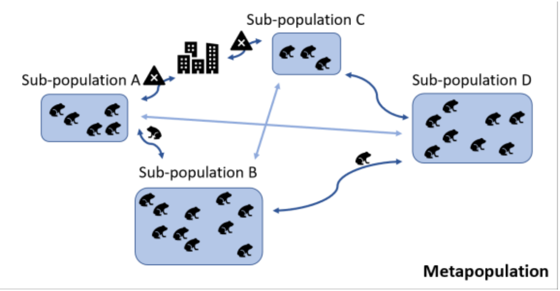
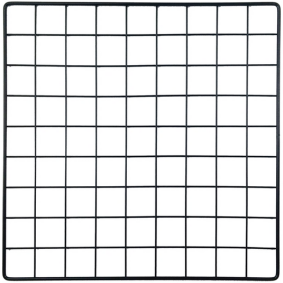


Figure: Metapopulations. Figure source: https://biol420eres525.wordpress.com/2019/04/12/application-of-metapopulation-theory-to-conservation/.

Lattice Models

- Basic idea: divide space up into discrete squares.
- Do SIR (or whatever) in each square.
- Add coupling/migration from square to square.
- (A lattice model can be thought of as a metapopulation model on a regular grid.)
- One can also simulate a discrete process on a grid



Individual-Based Models (aka Agent-Based Models)

- Simulate individuals directly instead of populations.
- IMO: Neither a scourge nor a panacea.

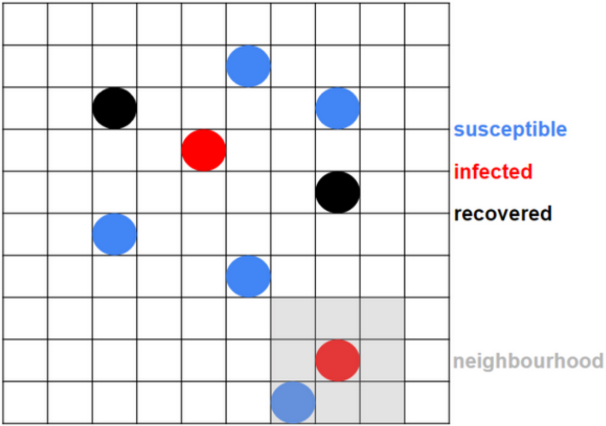


Figure: Source: https://www.supplychaindataanalytics.com/agent-based-sir-model-python-example/

Individual-Based Models (aka Agent-Based Models)

- Models can quickly become very complicated and difficult to understand.
- IBMs are an important and useful complement to other types of models.

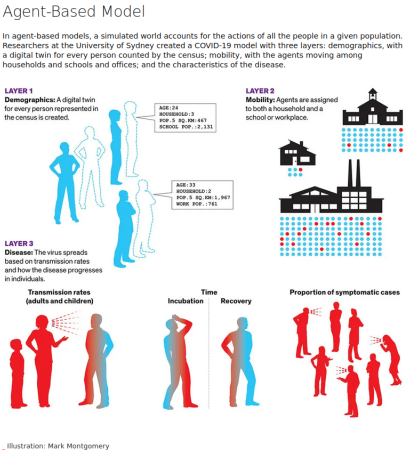


Figure: Source: https://spectrum.ieee.org/why-modeling-the-spread-of-covid19-is-so-damn-hard

Network Models

- Who has contact with whom?

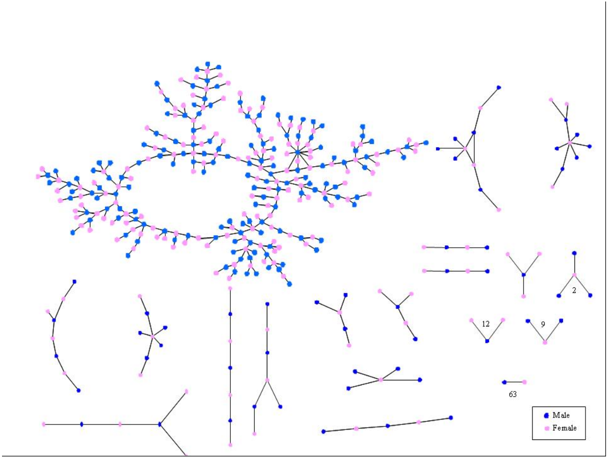


Figure: Source: Bearman, Peter S., James Moody, and Katherine Stovel. "Chains of affection: The structure of adolescent romantic and sexual networks." *American journal of sociology* 110.1 (2004): 44-91..

Network Models

- Can be very difficult to figure out who has contact with whom.
- Often one uses models (reasonable guesses) for network structure, since the network structure can't be directly determined.

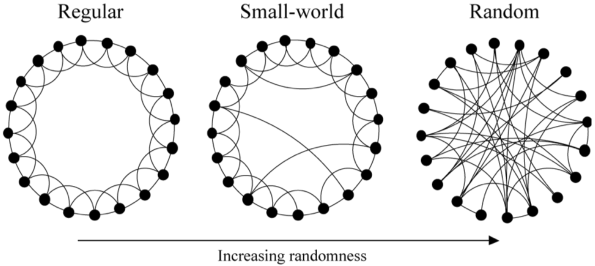


Figure: Small-world networks. Image source: https://chih-ling-hsu.github.io/2020/05/15/watts-strogatz.

Continuous Space, Continuous Time Models

- PDEs: Partial Differential Equations
- S, I, and R are now functions of t, x, and y.
- There is spatial diffusion and temporal SIR dynamics.

$$\frac{\partial S}{\partial t} = D_S \nabla^2 S - \beta S I \ , \tag{1}$$

$$\frac{\partial I}{\partial t} = D_I \nabla^2 I + \beta S I - \gamma I \ , \tag{2}$$

$$\frac{\partial R}{\partial t} = D_R \nabla^2 R + \gamma I \ , \tag{3}$$

where

$$\nabla^2 S = \frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2} \ . \tag{4}$$

The Ds are constants that determine the rate at which diffusion occurs.