Chapter 5.1 and 7.1 and 7.2 and 7.3

Linear Algebra with applications to differential equations

College of the Atlantic. Winter 2019

1. (Re)introduce yourself to your partners. We finally got a real snowstorm, eh? Only took until March.

2. Consider the differential equation

$$y'' - 5y' - 14y = 0 . \tag{1}$$

   (a) Find two solutions to the differential equation.

   (b) Calculate the Wronskian of the two solutions you found.

   (c) Find the solution to the differential equation that has $y(0) = 2$ and $y'(0) = 4$.

. . . .

3. Consider the first-order system:
$$x_1' = 4x_1 - 3x_2 , \tag{2}$$
$$x_2' = 6x_1 - 7x_2 . \tag{3}$$

   (a) Check to see which of the following vector functions $x(t)$ are solutions to the system:

$$\vec{x_a} = \begin{bmatrix} 3e^{2t} \\ 2e^{2t} \end{bmatrix} , \quad \vec{x_b} = \begin{bmatrix} e^{-5t} \\ 3e^{-5t} \end{bmatrix} , \quad \vec{x_a} = \begin{bmatrix} 2e^{4t} \\ 3e^{4t} \end{bmatrix} . \tag{4}$$

   (b) Find the solution to the system that satisfies

$$\vec{x}(0) = \begin{bmatrix} 3 \\ 0 \end{bmatrix} , \quad \vec{x}'(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} . \tag{5}$$

. . . .

4. Find the eigenvalues and eigenvectors for the matrix A:

$$A = \begin{bmatrix} 4 & -3 \\ 6 & -7 \end{bmatrix} . \tag{6}$$

. . . .

5. Find the general solution to the following linear system:

$$x_1' = 4x_1 + 2x_2 , \tag{7}$$

$$x_2' = 3x_1 - x_2 . \tag{8}$$