Chapter C9: Rotational Energy

Physics I

College of the Atlantic

This chapter is about how to account for energy when things are spinning. There's not really any new fundamental physics in this chapter. The main idea is that from the definition of angle, Eq. (1) below, we can figure out how to expand our energy bookkeeping to conveniently keep track of the energy of rotating objects.

C9.1: Introduction to Rotational Energy

C9.2: Measuring Angles

You will use radians and you'll like them. Really.

An angle θ is defined by:

$$|\theta| \equiv \frac{s}{r} \ . \tag{1}$$

This formula gives the angle in *radians*, not degrees.

C9.3: Angular Velocity

Definitions and Equations:

$$\omega \equiv \left|\frac{d\theta}{dt}\right| \ . \tag{2}$$

Since $s = r\theta$,

$$ds = r|d\theta| \ . \tag{3}$$

Thus,

$$v = \frac{ds}{dt} = \frac{r|d\theta|}{dt} = r\omega \ . \tag{4}$$

Example:

What is the angular velocity of the second hand on the clock?
If the second hand is 25 cm long, what is the speed of the end of the hand?

C9.4: The moment of inertia

Equation C9.7 gives the central idea of this chapter:

$$K^{\rm rot} = \frac{1}{2} I \omega^2 \ , \tag{5}$$

where I, the *moment of inertia*, is given by:

$$I \equiv \sum_{i=1}^{N} m_i r_i^2 \ . \tag{6}$$

C9.5: Calculating Moments of Inertia

Don't worry about this section. The main idea is that we can use Eq. (6) to determine the moments of inertia of various objects.

C9.6: Translation and Rotation

The total kinetic energy of an object that is rotating and moving is given by:

$$K = K^{\rm cm} + K^{\rm rot} = \frac{1}{2} M v_{\rm cm}^2 + \frac{1}{2} I \omega^2 \ , \tag{7}$$

C9.7: Rolling without Slipping

If an object rolls without slipping at a velocity $v_{\rm cm}$, then:

$$\omega = \frac{v_{\rm cm}}{R} \ . \tag{8}$$

Example: A .5 kg plate with a radius of 10 cm rolls without slipping at 1 m/s. What is the plate's total kinetic energy?