Summary of Unit Five

Proper Time:

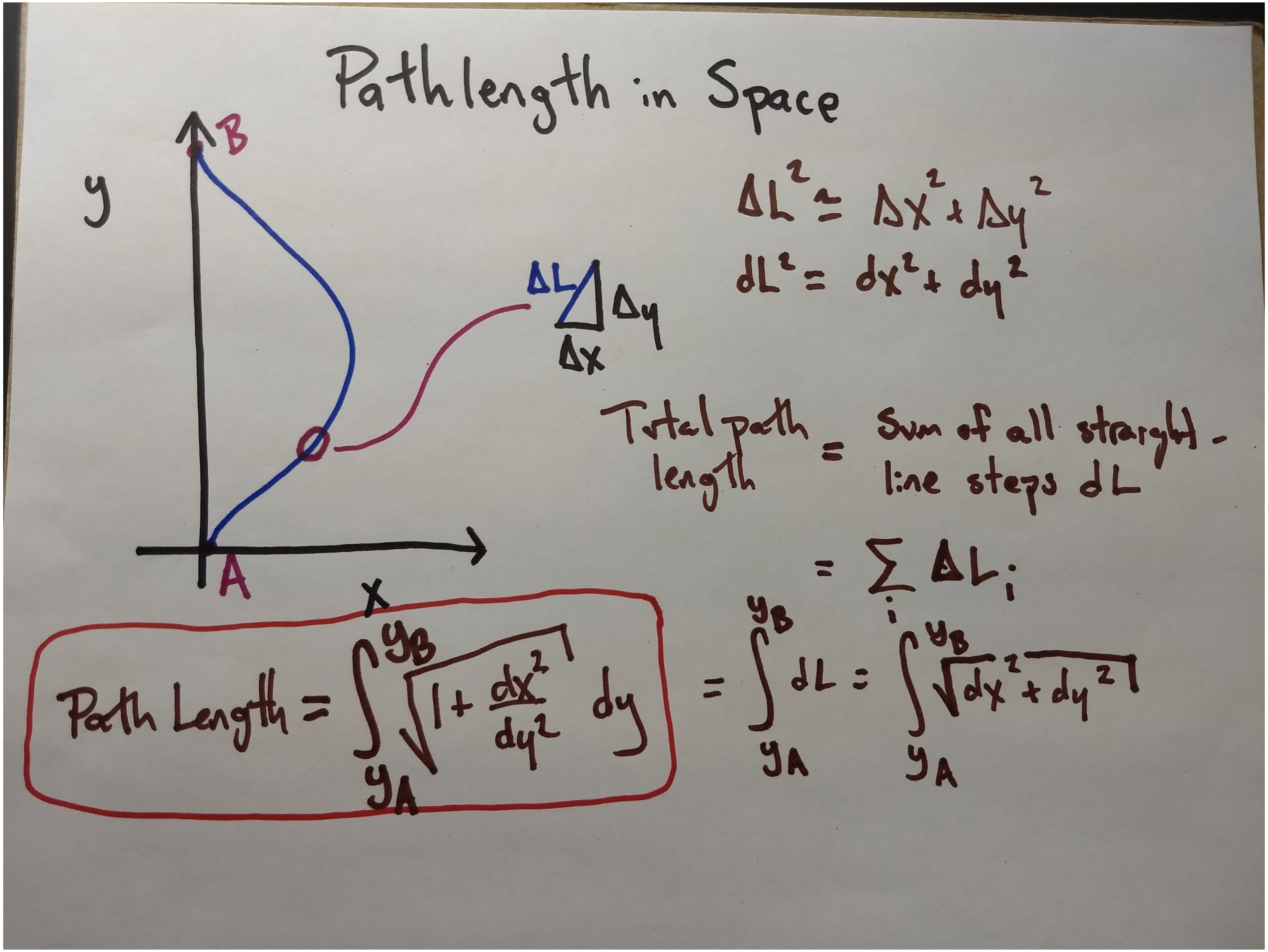
Physics II

Special Relativity

David P. Feldman

http://tiny.cc/RelativityAtCOA

Pathlength in Space



Pathlength in Space

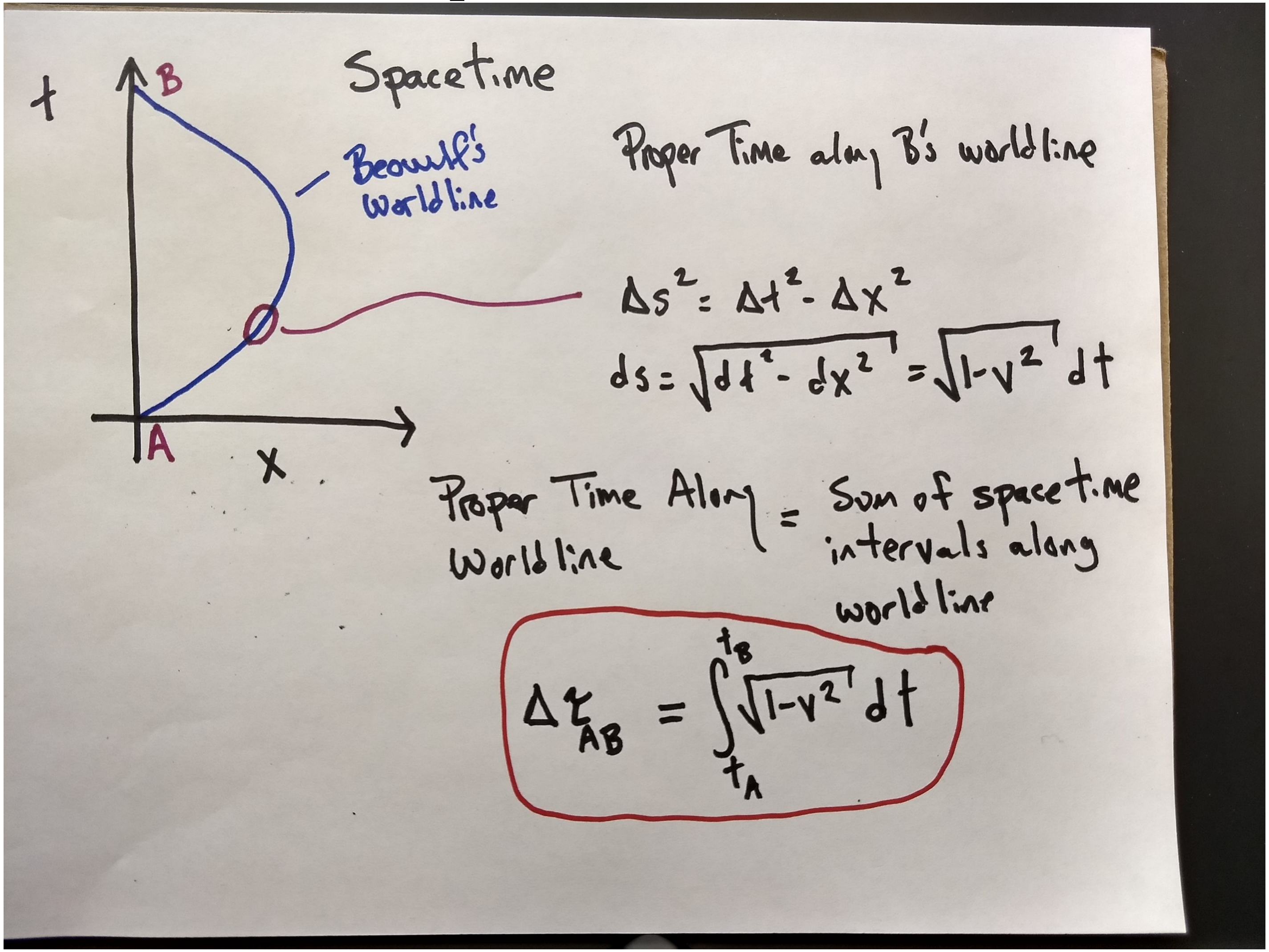
$$\text{Pathlength} = \int_{y_A}^{y_B} \sqrt{1 + (\frac{dx}{dy})^2}\, dy \ .$$

- But in the special case where dx/dy is constant, this equation simplifies to:

$$\text{Pathlength} = \sqrt{1 + (\frac{dx}{dy})^2}\, \Delta y \ .$$

- Geometrically, dx/dy being constant means that the path is a straight line.

Proper Time and Worldlines in Spacetime



Proper Time and Worldlines in Spacetime

$$\text{ProperTime} = \Delta\tau = \int_{t_A}^{t_B} \sqrt{1-v^2}\, dt\ .$$

- In the special case in which the speed (but not necessarily velocity) is constant:

$$\text{ProperTime} = \Delta\tau = \sqrt{1-v^2}\, \Delta t\ .$$

- Note that Delta t must be a coordinate time.

Binomial Approximation

$$(1+x)^a \approx 1+ax \text{ if } x \ll 1 \, .$$

- Super useful and commonly used.
- For us in this unit:

$$\sqrt{1-v^2} = (1-v^2)^{\frac{1}{2}} \approx 1 - \frac{1}{2}v^2; \text{ if } v \ll 1 \, .$$

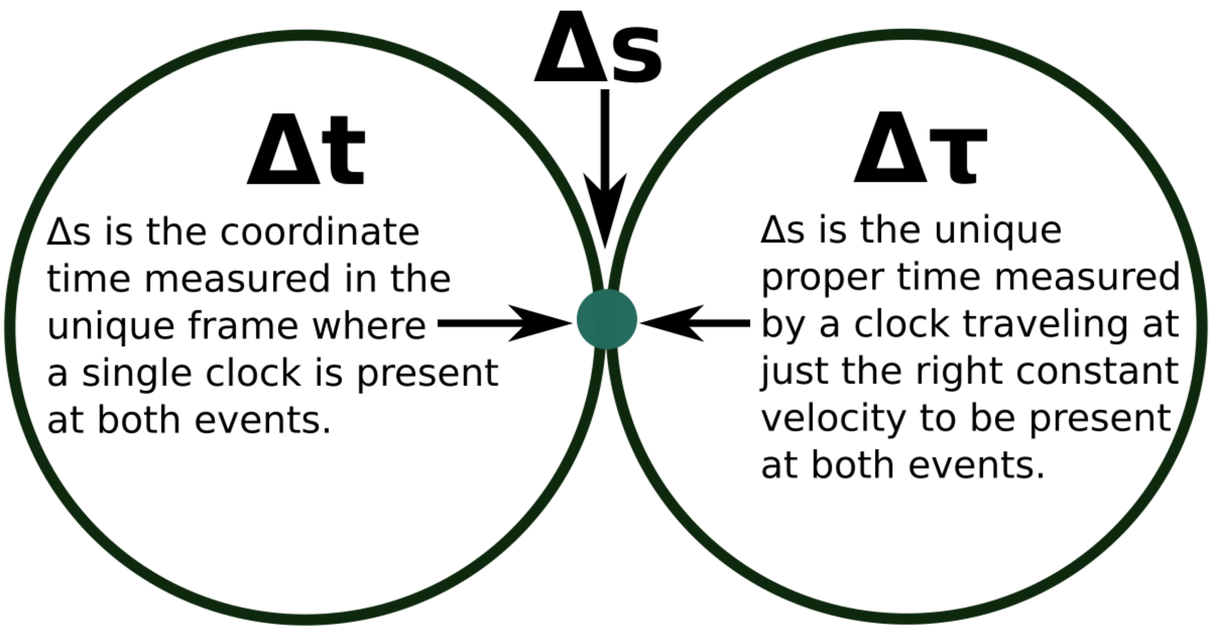
- And:

$$\Delta\tau = \sqrt{1-v^2}\Delta t \approx (1-\frac{1}{2}v^2)\Delta t = \Delta t - \frac{1}{2}v^2\Delta t$$

- So, diff between coordinate and proper time

$$|\Delta\tau - \Delta t| \approx \frac{1}{2}v^2\Delta t$$

Relations Among Time Intervals



* **Δt: Coordinate time** is the time between two events measured in an inertial reference frame by a pair of synchronized clocks.
* **Δτ: Proper time** is a time between two events as measured by a single clock present at both events. Depends on worldline of the clock.
* **Δs: The spacetime interval** is the time between two events as measured by an inertial clock present at both events. Unique.

Figure based on Fig.R3.9, Tom Moore, Six Ideas that Shaped Physics: Unit R. (2003)

Relations Among Time Intervals

$$\Delta t \geq \Delta s \geq \Delta \tau$$

- Note that this is backward compared to intervals in space:

$$\Delta y \leq \Delta L \leq \text{Path Length}$$

- The reason for this is that the distance formula and the metric equation lead to different geometries
- Space: $\Delta L^2 = \Delta x^2 + \Delta y^2$.
- Spacetime: $\Delta s^2 = \Delta t^2 - \Delta x^2$.