The Postulates[1] of Quantum Mechanics

Physics II: Modern Physics

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The first postulate of quantum mechanics:

1. The state of a quantum mechanical system is described mathematically by a normalized ket $|\psi\rangle$ that contains all the information we can know about the state.

Operators: Operators are mathematical objects that operate on kets and return a new ket. For example, an operator $\hat{A}$:

$$\hat{A}|\psi\rangle = |\phi\rangle \ . \tag{1}$$

You can think of an operator as being like a function that takes a ket as input and returns a ket as output. In general, the output ket is different than the input ket.

However, for a given operator, there are some special kets, or vectors, that do not change direction when the operator acts on them. These vectors are known as *eigenvectors*. For example, if

$$\hat{A}|\psi_n\rangle = a_n|\psi_n\rangle \ . \tag{2}$$

Here a_n is known as the *eigenvalue*.

Observables and Measurement: We can now restate postulate four in more general terms and state three new postulates of quantum mechanics.

2. A physical observable is described mathematically by an operator that acts on kets.

3. The only possible result of a measurement of an observable is one of the eigenvalues a_n of the corresponding operator $\hat{A}$.

4. The probability of obtaining the eigenvalue a_n in a measurement of the observable A when the system is in state $|\psi\rangle$ is:

$$\text{Prob of a}_\text{n} = |\langle a_n|\psi\rangle|^2 \ . \tag{3}$$

5. After measurement of $\hat{A}$ that yields a_n, the quantum system is in a new state that is the normalized eigenvector corresponding to the eigenvalue a_n.

Note that measurement changes the state of the system, unless the system happens to already be in an eigenstate of the operator. This is known as *the collapse of the wavefunction* or state reduction or state projection.

[1] These postulates are those listed by McIntyre in *Quantum Mechanics: A Paradigms Approach*, Pearson, 2012. These postulates are quite standard, but they often aren't explicitly enumerated.

As an example of these postulates, consider the $\hat{S}_z$ operator corresponding to the z-component of the spin. The eigenvalues for this operator are $+1$ and -1, since we know that these are the two possible outcomes for such a measurement. The two eigenvectors we've denoted $|+\rangle$ and $|-\rangle$. In general, the eigenvectors can (essentially) always serve as a basis.

In matrix representation, the three components of the spin-1/2 operator are:

$$\hat{S}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} , \tag{4}$$

$$\hat{S}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \tag{5}$$

$$\hat{S}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} . \tag{6}$$

These might be useful for some optional problems.

Incompatible Observables and Non-Commuting Operators: Two operators *commute* if they yield the same thing regardless of their order of operation. For example, addition is commutative. However, the spin operators above do not commute.

If operators $\hat{A}$ and $\hat{B}$ commute, then we can show that they have the same eigenvectors. This, in turn, means that the measurement of one will not "un-do" the measurement of the other.

Quantum Time Evolution: Quantum mechanics also describes how the state $|\psi\rangle$ changes in time. This forms the sixth, and final, postulate:

6. The time evaluation of a quantum system is determined by:

$$i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle , \tag{7}$$

   where $\hat{H}$ is the Hamiltonian, or total energy operator, for the system.

Equation (7) is the famous Schrödinger equation. It is, roughly speaking, the equivalent of Newton's Second Law for quantum mechanics.

Note that the equation describing $|\psi(t)\rangle$ is deterministic, just like Newton's laws or the Maxwell equations for electricity and magnetism. Probabilities and wave functions collapsing on occur when measurements are made, not when $\psi(t)$ just does its own thing according to the Schrödinger equation. Note that Eq. (7) is deterministic. The wavefunction ψ evolves in time deterministically; the indeterminism in quantum mechanics is associated with measurement.