Chapter E5: Voltage

E5.2 Resistance and Ohm's Law

Two important ideas from previous chapters:

$$|\Delta\mathcal{V}_b| \;=\; |\Delta\mathcal{V}_{\rm wire}| \;=\; E_{\rm wire}L \;, \tag{1}$$

and

$$\vec{J} \;=\; \sigma\vec{E} \;. \tag{2}$$

We can use this to relate the current $I = JA$ to the voltage difference of a battery. We define the resistance of an object by:

$$R \;\equiv\; \frac{|\Delta\mathcal{V}|}{I} \;. \tag{3}$$

If the R of some object is constant as I varies, then we say that the object is *ohmic*. Note: many objects aren't ohmic.

For a wire of length L, uniform thickness A and constant conductivity σ,

$$R \;=\; \frac{L}{\sigma A} \;. \tag{4}$$

E5.3 Two Wires in Series

For objects in a circuit in series,

$$I \;=\; I_1 \;=\; I_2 \;=\; I_3 \;\ldots\; , \tag{5}$$

and

$$|\Delta\mathcal{V}_b| \;= |\Delta\mathcal{V}_1| + |\Delta\mathcal{V}_2| + |\Delta\mathcal{V}_3| + \ldots \;, \tag{6}$$

where I_n is the current through the $n^{\rm th}$ object and $|\Delta\mathcal{V}_n|$ is the potential difference across the $n^{\rm th}$ object. You should understand why these equations have to be true.

E5.4 Total Resistance in a Series Circuit

An immediate consequence of the above two equations is that the total resistance in a series circuit is given by:

$$R_{\rm tot} \;=\; R_1 + R_2 + R_3 + \ldots \;, \tag{7}$$

where R_n is the resistance of the n^{th} object.

E5.5 The Voltage at a Point

To speak about the voltage at a point, as opposed to the voltage difference between two points, it is necessary to fix some reference point as $\mathcal{V} = 0$.

E5.7 Electrical Power

The electrical power dissipated in a circuit is given by

$$P = |\Delta\mathcal{V}|I \; . \tag{8}$$

The unit of power is the Watt (1 W = 1J/s). A frequently-used unit of energy is the kilowatt hour.

Examples:

1. Your bathtub holds .8 cubic meters of water. It takes 10 minutes to fill up the tub. Assuming a constant flow rate, what is the current (of water) into the tub? If the round faucet is 5cm long and has a radius of 1 cm, what is the current density?