

Warm-Up Problems: Pigeonholes

Proofs & Mathematical Structures

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Figure 1: Pigeons in holes. Image by Wikipedia users BenFrantzDale and McKay. Image source <https://commons.wikimedia.org/wiki/File:TooManyPigeons.jpg>. Licensed under the Creative Commons Attribution-Share Alike 3.0 Unported license.

1. You have five pairs of socks, each of which is a different color. All of the socks are mixed together in a drawer. You begin to grab socks out of the drawer one at a time. How many socks do you have to have grabbed before you are guaranteed to have at least one matching pair?
2. Given any five numbers from the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$, prove that at least two of the numbers will add up to 9.
3. Prove the following statement: Consider a 3×3 square. If you choose ten points at random on the interior of the square, then there is at least one pair of points which are at most a distance $\sqrt{2}$ from each other.
4. What does Fig. 1 have to do with these problems?