

Here are some problems for Monday, 1 May, 2023.

1. Prove the following proposition. For all  $n \in \mathbb{N}_0$ ,  $1 + 2 + 4 + \cdots + 2^n = 2^{n+1} - 1$ .

2. Prove that:

$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \cdots + \frac{n}{(n+1)!} = 1 - \frac{1}{(n+1)!} . \quad (1)$$

3. Prove that for all  $n \in \mathbb{N}$ ,  $n^2 - n$  is even. Do this proof three ways:

(a) Consider two cases:  $n$  is odd and  $n$  is even.

(b) Use induction.

(c) Use Proposition 4.2.

4. Find a formula for the sum:

$$2 + 4 + 6 + \cdots + 2n . \quad (2)$$

Prove that your formula is correct two ways:

(a) Use Proposition 4.2.

(b) Use induction.