Homework assignment three

1. For the graph in Fig. 1, do the following. Be sure to state the definition of all quantities you’re calculating.

   (a) Find the average cluster coefficient $C$.
   (b) Compute the degree distribution $P(k)$.
   (c) Compute the average degree $\langle k \rangle$.
   (d) Find the average path length $\ell$.

2. For the graph of Fig. 1, write down the adjacency matrix. Are there different matrices that describe the same graph? Discuss.

3. Consider a regular graph that is a four by four grid. Assume that the grid is wrapped around on itself so that the right most nodes are neighbors with the left most nodes, and the top nodes are neighbors with the bottom nodes. This shape turns out to be a donut. For this graph, do the following:

   (a) Write down the adjacency matrix.
   (b) Calculate the cluster coefficient
   (c) Calculate the mean path length $\ell$.
   (d) Calculate the degree distribution.
   (e) Calculate the average degree $\langle k \rangle$.

   If this problem seems too easy, it probably means you’re doing it correctly.

4. Use proof by induction to prove the familiar power rule for derivatives.

$$\frac{d}{dx} x^n = nx^{n-1},$$  \hspace{1cm} (1)

where $n$ is any integer greater than zero. Feel free to consult a calculus textbook. However, you should cite your sources.
Figure 1: A graph

5. Consider the subset of the natural numbers consisting of all perfect squares: \{1, 4, 9, 16, \ldots\}. Argue that this is a countably infinite set. In your argument, be sure to state what a countably infinite set is.

6. There are many systems that could be viewed as a network. Come up with an example that we haven’t talked about much in class, and discuss. Mention both the benefits and drawbacks of looking at the system as a network. Your response should probably be around two or three paragraphs. I would prefer a typed response, unless your handwriting is unusually neat.